Reduced models for domain walls in soft ferromagnetic films

Lukas Döring

Conference on Nonlinearity, Transport, Physics, and Patterns

Fields Institute, Toronto

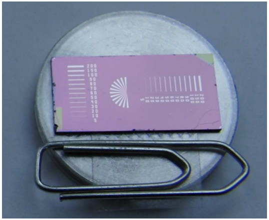
06/10/14

RWTH AACHEN UNIVERSITY

Max Planck Institute for Mathematics in the Sciences

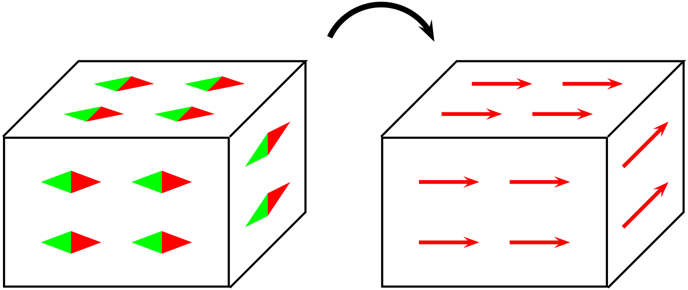
MAX-PLANCK-GESELLSCHAFT

Modelling ferromagnetic thin films



$\Omega \subset \mathbb{R}^3$ sample

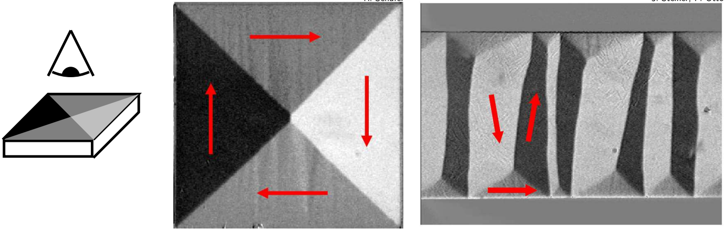
$m\colon \Omega \to \mathbb{S}^2$ magnetization



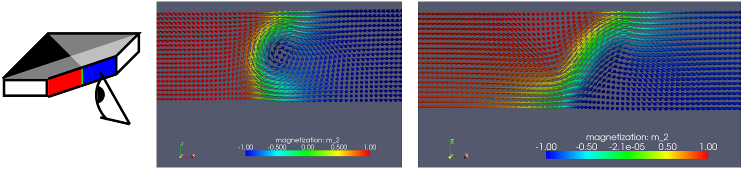
“Elementary magnets” $\rightsquigarrow$ Unit-length vector field

Magnetization patterns in thin-film ferromagnets

R. Schäfer   J. Steiner, F. Otto



Magnetization patterns in Permalloy films



Numerical simulation of domain walls

Landau-Lifshitz (free) energy

Observed patterns: Local minimizers $m\colon \Omega \subset \mathbb{R}^3 \to \mathbb{S}^2$ of

$$E(m) = d^2 \int_\Omega |\nabla m|^2 dx \quad \text{Exchange energy}$$

$$+ \int_{\mathbb{R}^3} |h_{\text{str}}|^2 dx \quad \text{Stray-field energy} \quad \begin{cases} \nabla \cdot (h_{\text{str}} + \mathbf{1}_\Omega m) = 0 \\ \nabla \times h_{\text{str}} = 0 \end{cases}$$

$$+Q \int_\Omega 1 - (e \cdot m)^2 dx \quad \text{Anisotropy energy for } e \in \mathbb{S}^2,\ Q \ll 1$$

$$-2 \int_\Omega h_{\text{ext}} \cdot m \, dx \quad \text{Zeeman energy}$$

Well-accepted   Non-convex   Non-local

Landau-Lifshitz (free) energy

Observed patterns: Local minimizers $m\colon \Omega \subset \mathbb{R}^3 \to \mathbb{S}^2$ of

$$E(m) = d^2 \int_\Omega |\nabla m|^2 dx \quad \text{Exchange energy}$$

$$+ \int_{\mathbb{R}^3} |h_{\mathrm{str}}|^2 dx \quad \text{Stray-field energy} \quad \begin{cases} \nabla \cdot (h_{\mathrm{str}} + \mathbf{1}_\Omega m) = 0 \\ \nabla \times h_{\mathrm{str}} = 0 \end{cases}$$

$$+Q \int_\Omega 1 - (e \cdot m)^2 dx \quad \text{Anisotropy energy for } e \in \mathbb{S}^2,\ Q \ll 1$$

$$-2 \int_\Omega h_{\mathrm{ext}} \cdot m \, dx \quad \text{Zeeman energy}$$

Well-accepted Non-convex Non-local

Landau-Lifshitz (free) energy

Observed patterns: Local minimizers $m\colon \Omega \subset \mathbb{R}^3 \to \mathbb{S}^2$ of

$$E(m) = d^2 \int_\Omega |\nabla m|^2 dx \quad \text{Exchange energy}$$

$$+ \int_{\mathbb{R}^3} |h_{\mathrm{str}}|^2 dx \quad \text{Stray-field energy} \quad \begin{cases} \nabla \cdot (h_{\mathrm{str}} + \mathbf{1}_\Omega m) = 0 \\ \nabla \times h_{\mathrm{str}} = 0 \end{cases}$$

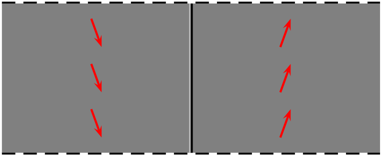
$$+Q \int_\Omega 1 - (e \cdot m)^2 dx \quad \text{Anisotropy energy for } e \in \mathbb{S}^2,\ Q \ll 1$$

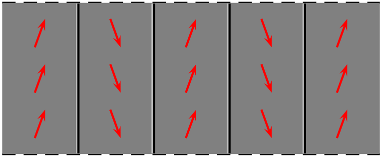
$$-2 \int_\Omega h_{\mathrm{ext}} \cdot m \, dx \quad \text{Zeeman energy}$$

Well-accepted Non-convex Non-local

Outline

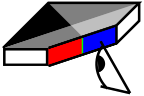
Single wall in infinitely extended film

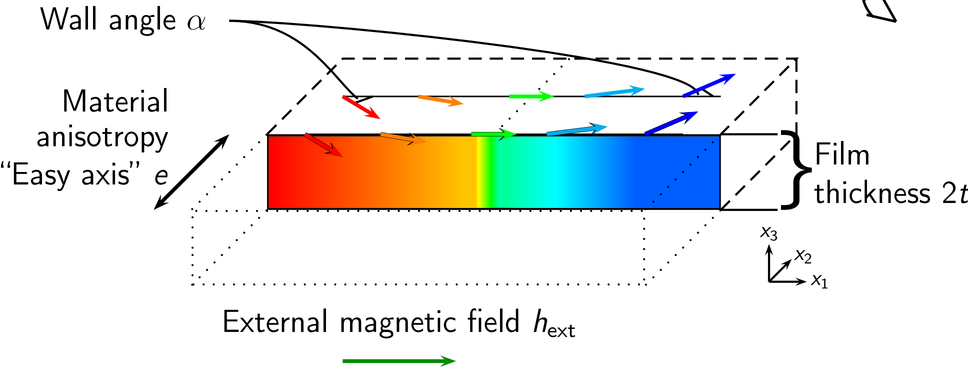


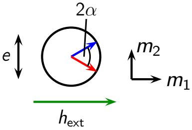


Periodic domain pattern with interacting wall tails

Wall patterns on cross-section of film



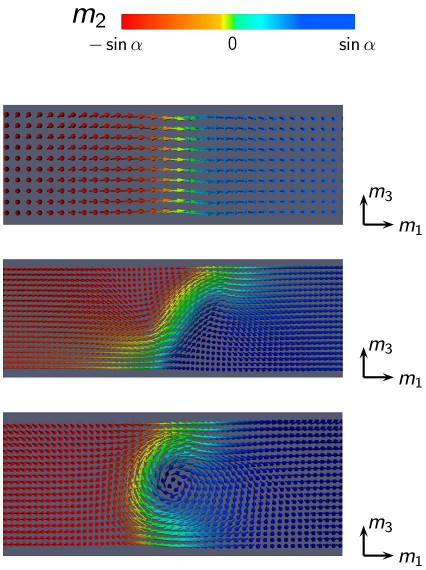




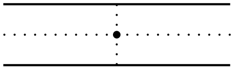
Anisotropy Q and external field h_{ext} determine wall angle α.

Wall angle α and film thickness t determine wall type.

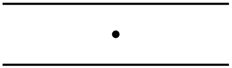
Three wall types



Symmetric Néel wall

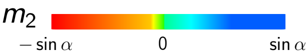


Asymmetric Néel wall

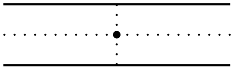


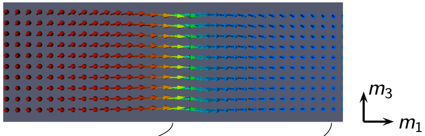
Asymmetric Bloch wall

Three wall types



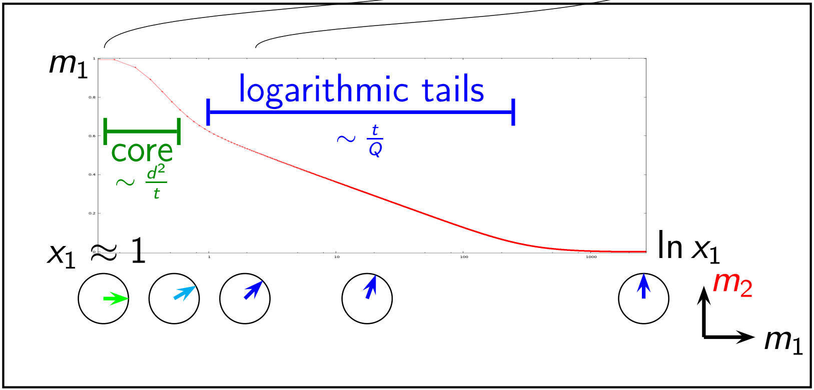
Symmetric Néel wall



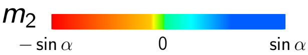


Asy

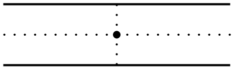
Asy

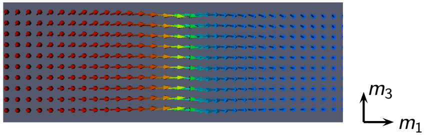


Three wall types

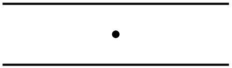


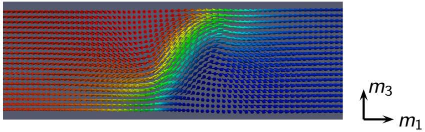
Symmetric Néel wall



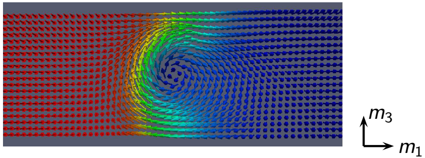


Asymmetric Néel wall



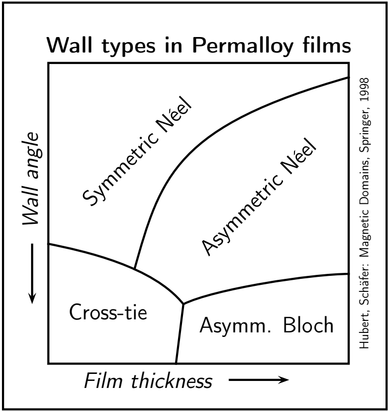


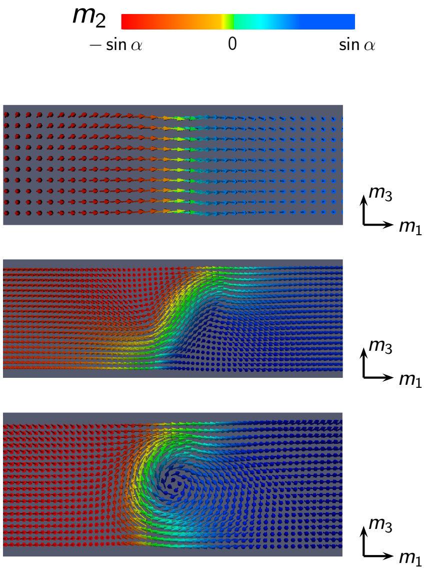
Asymmetric Bloch wall



Aim: Understand transitions between wall types for $Q \ll 1$

Three wall types





Aim: Understand transitions between wall types for $Q \ll 1$

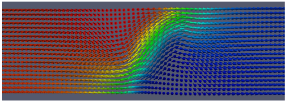
The critical regime: Optimal mix . . .

$$\min_{\substack{m \text{ wall of} \\ \text{angle } \frac{\pi}{2}}} E_{2D}(m) \overset{\text{Otto, '02}}{\sim} \begin{cases} t^2 \ln^{-1} \frac{t^2}{d^2 Q}, & \text{if } \frac{t^2}{d^2} \ll \ln \frac{1}{Q}, \\ d^2, & \text{if } \frac{t^2}{d^2} \gg \ln \frac{1}{Q}. \end{cases}$$

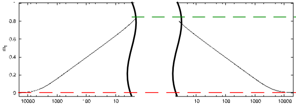
What happens in critical regime: $\frac{t^2}{d^2} = \lambda \ln \frac{1}{Q}$?

Optimal wall profile for angle α =

asymm. "$2\frac{1}{2}$-d" core

$m = \begin{pmatrix} \cos\theta \\ -\sin\theta \\ 0 \end{pmatrix}$  $m = \begin{pmatrix} \cos\theta \\ +\sin\theta \\ 0 \end{pmatrix}$ +

long-range 1-d tails



$m_1 = \cos\theta$

$m_1 = \cos\alpha$

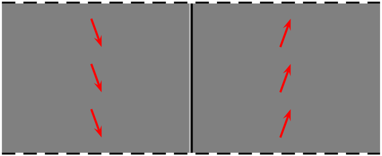
Optimal mix: θ $\qquad$ $\alpha - \theta$

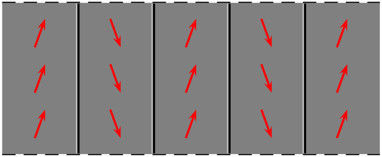
Quantification of optimal mix difficult to access by brute-force numerics.

. . . of core and tails

Outline

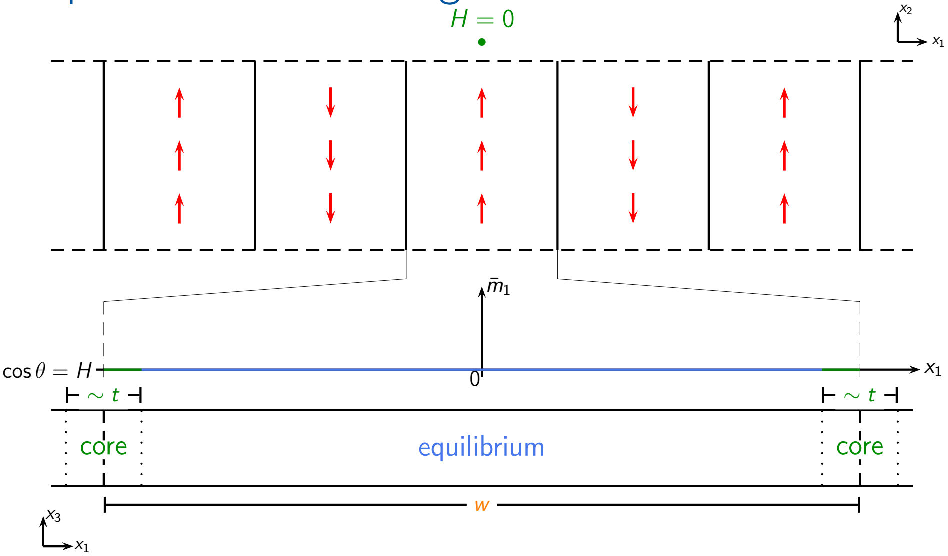
Single wall in infinitely extended film





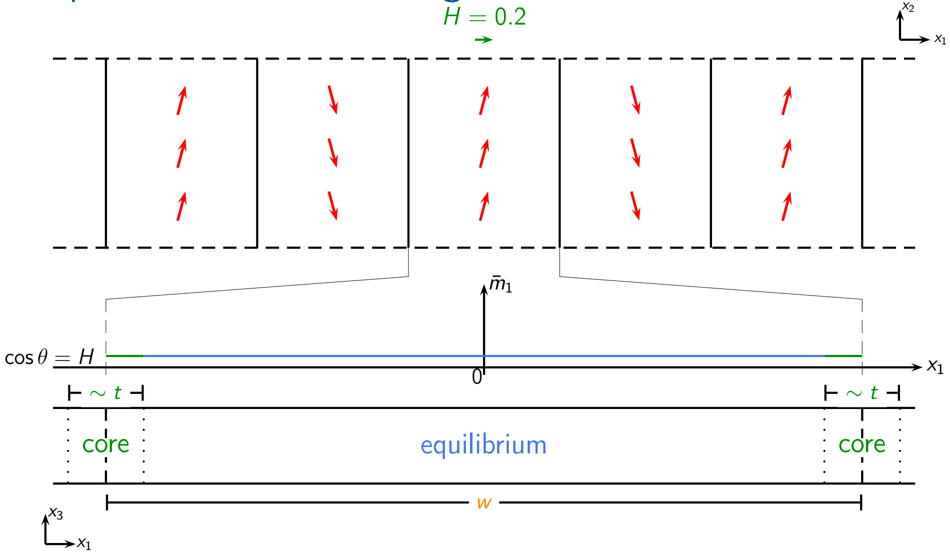
Periodic domain pattern with interacting wall tails

Expected behavior: Large domain width...



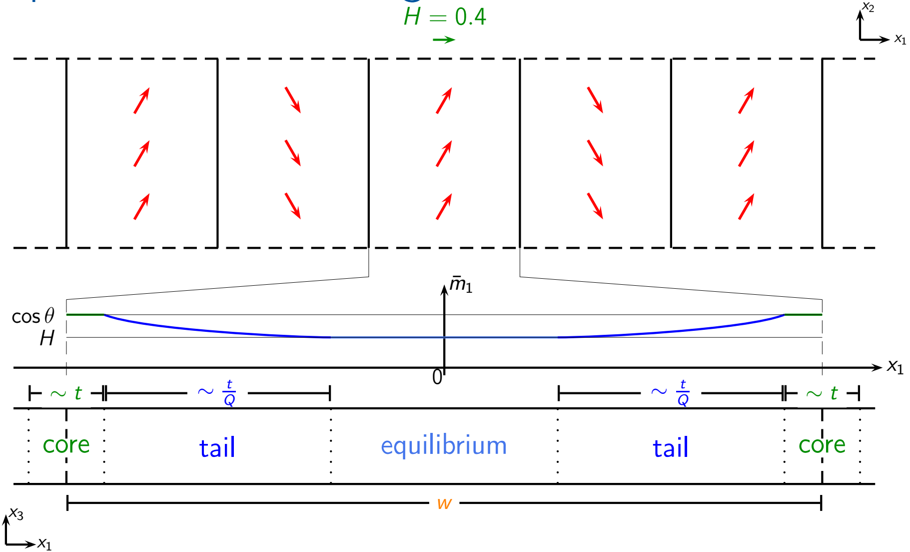
...similar to one-wall case

Expected behavior: Large domain width. . .



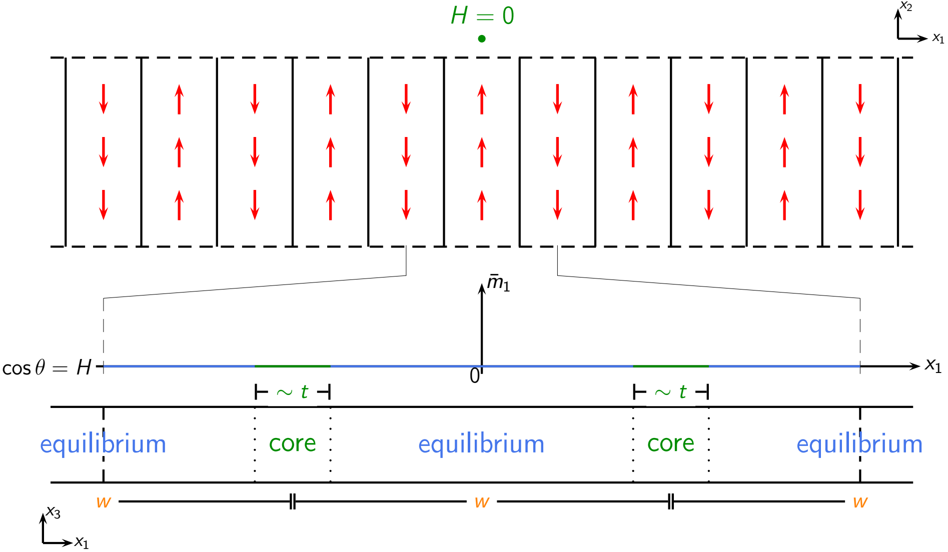
. . . similar to one-wall case

Expected behavior: Large domain width...



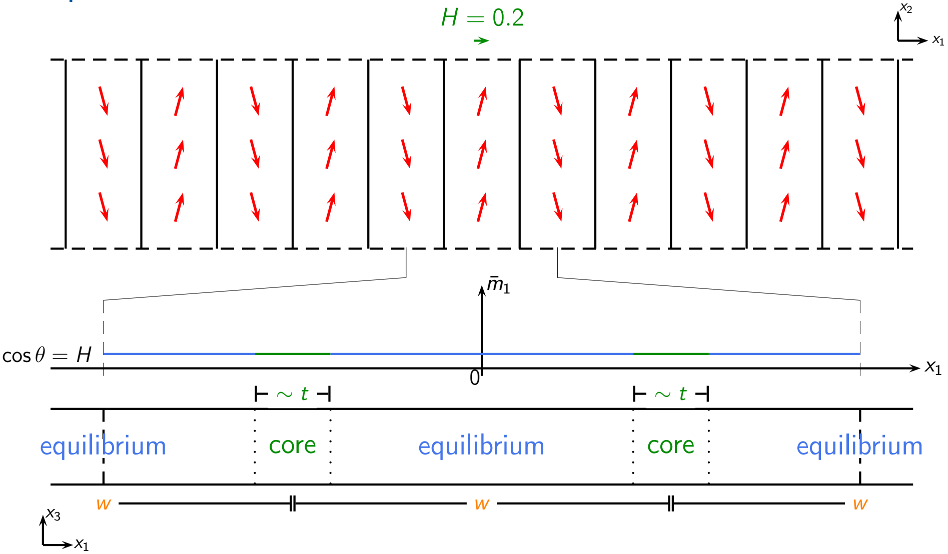
...similar to one-wall case

Expected behavior: Small domain width...



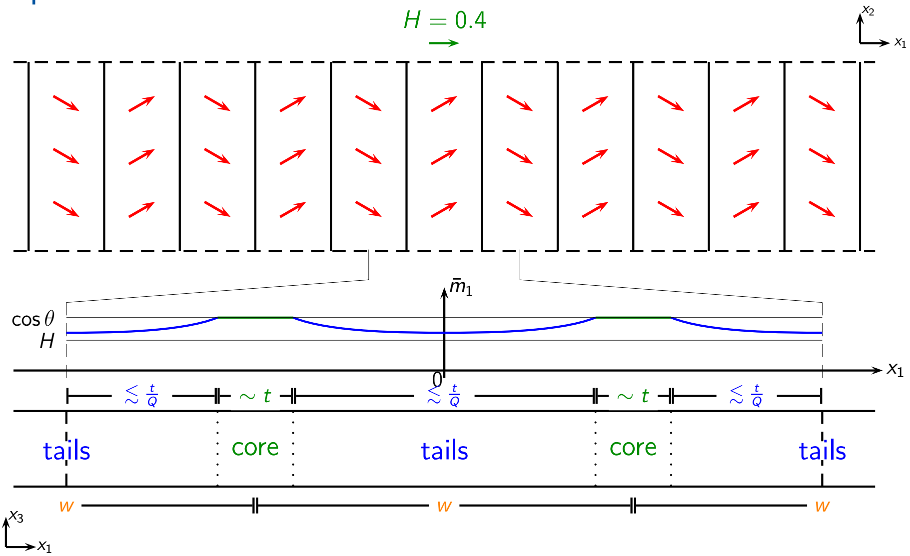
...leads to coalescing tails

Expected behavior: Small domain width...



... leads to coalescing tails

Expected behavior: Small domain width...

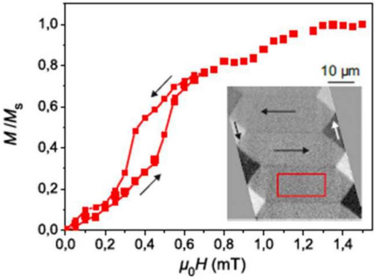


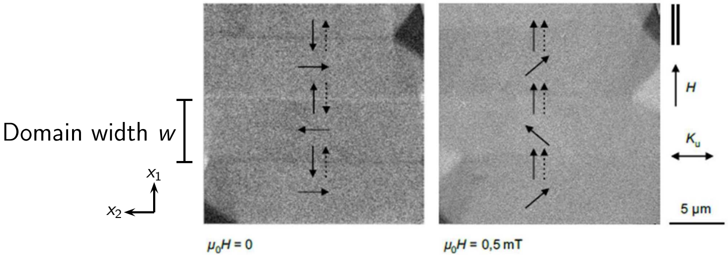
...leads to coalescing tails

Strongly hysteretic transition between asym. walls. . .

Elongated CoFeB elements
$2t = 120\text{nm},\ Q = 1.55 \cdot 10^{-3}$,
$d = 3.86 \pm 0.3\text{nm}$.

Origin of large jump in
hard-axis magnetization?

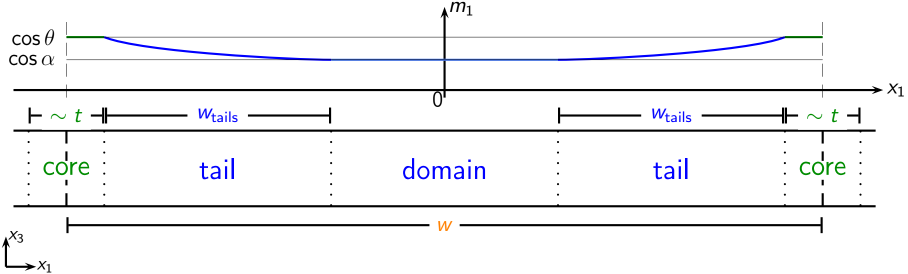




C. Hengst, IFW Dresden

. . . due to interacting tails?

Just a few building blocks...



$$E_{2D} = \text{Exchange energy} + \text{Stray-field energy} + \text{Bulk energy}$$

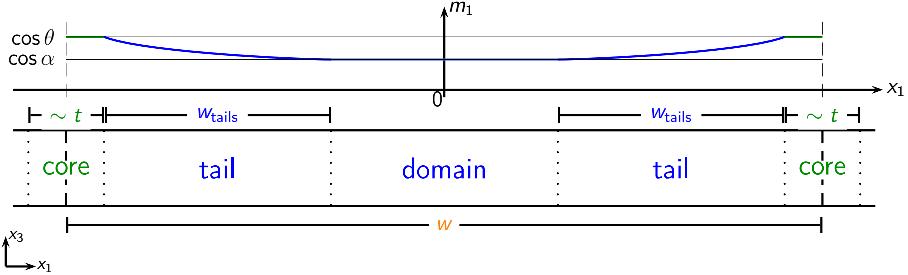
$$\approx d^2 \int |\nabla m_\theta^{\text{core}}|^2 dx + 2t^2 \int \left| \left|\tfrac{d}{dx_1}\right|^{\frac{1}{2}} m_1^{\text{tails}} \right|^2 dx_1$$

$$+ 2Qwt \fint (m_1^{\text{tails}} - H)^2 dx_1, \quad \text{with } \left(\tfrac{t}{d}\right)^2 = \lambda \ln \tfrac{1}{Q}.$$

Interesting regime: $Qwt = \kappa\lambda\, d^2$; optimal $w_{\text{tails}} = \frac{w}{2}$.

...combined in an optimal way

Just a few building blocks. . .



$$E_{2D} = \text{Exchange energy} + \text{Stray-field energy} + \text{Bulk energy}$$

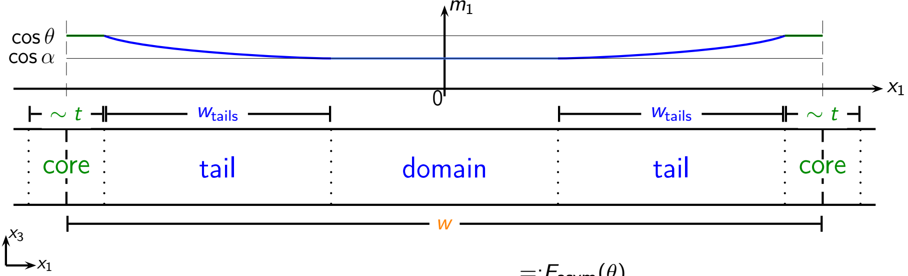
$$\approx d^2\Big(\int |\nabla m_\theta^{\text{core}}|^2 dx + 2\pi\lambda\,(\cos\theta - \cos\alpha)^2$$

$$+\,2\,\kappa\lambda\,(\cos\alpha - H)^2\Big), \quad \text{with } \left(\tfrac{t}{d}\right)^2 = \lambda \ln\tfrac{1}{Q}.$$

Interesting regime: $\frac{w}{t} = \frac{\kappa}{Q \ln \frac{1}{Q}}$; optimal $w_{\text{tails}} = \frac{w}{2}$.

. . . combined in an optimal way

Just a few building blocks...



$$d^{-2}\min_m E_{2D}(m) \approx \min_\theta\Bigg(\overbrace{\min_{\substack{m \text{ stray-field free}\\ \text{wall of angle } \theta}} \int |\nabla m|^2 dx}^{=:E_{\text{asym}}(\theta)}$$

$$+ 2\pi\lambda \min_\alpha\Big((\cos\theta - \cos\alpha)^2 + \tfrac{\kappa}{\pi}(\cos\alpha - H)^2\Big)\Bigg)$$

as $Q \to 0$, for $(\frac{t}{d})^2 = \lambda \ln \frac{1}{Q}$, $w = \kappa \frac{t}{Q \ln \frac{1}{Q}}$.

...combined in an optimal way

Reduced model for the structure of domain walls

Theorem ($\kappa = \infty$: D., Ignat, Otto; $\kappa < \infty$: D.)

There exist critical points m_Q of E_{2D}, such that for $Q \to 0$, λ the relative film thickness, κ the relative domain width:

$$d^{-2} E_{2D}(m_Q) \approx \min_{\theta \in [0, \frac{\pi}{2}]} \left(E_{asym}(\theta) + 2\pi\lambda \frac{\kappa}{\pi+\kappa} (\cos\theta - H)^2 \right)$$

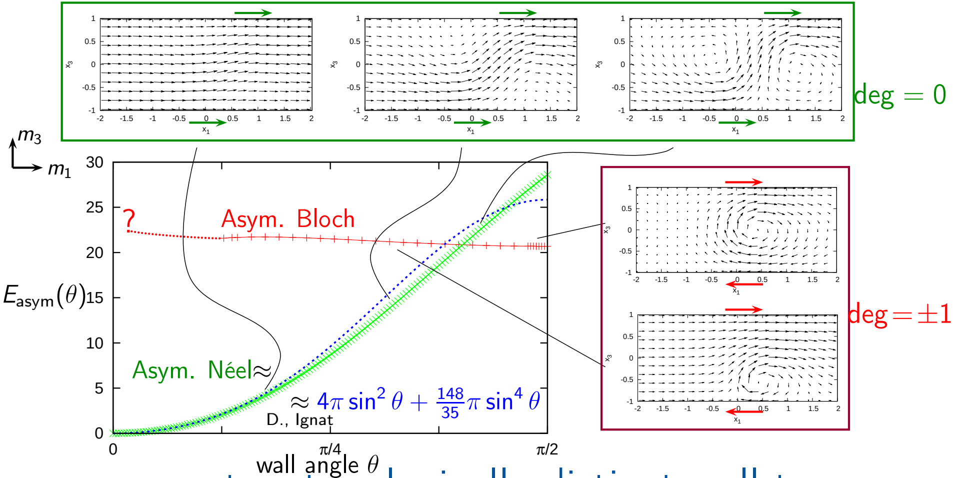
and

$$⨍_{domain} m_{1,Q}\, dx \approx \cos\alpha_{opt} = H + \frac{\pi}{\pi+\kappa} (\cos\theta_{opt} - H).$$

- Proof via Γ-conv. (minimize E_{2D} over periodic m).
- Compactness requires "shifting argument" to ensure that $\{m_Q\}_Q$ converges to a *domain wall*.

Stray-field free core: Néel and Bloch...

$$E_{\text{asym}}(\theta) := \min\left\{ \int_\Omega |\nabla m|^2 dx \;\middle|\; \begin{array}{c} m\colon \Omega \to \mathbb{S}^2 \text{ has wall angle } \theta, \\ \text{with } \nabla \cdot m' = 0 \text{ in } \Omega,\ m_3 = 0 \text{ on } \partial\Omega \end{array} \right\}$$



...two topologically distinct wall types

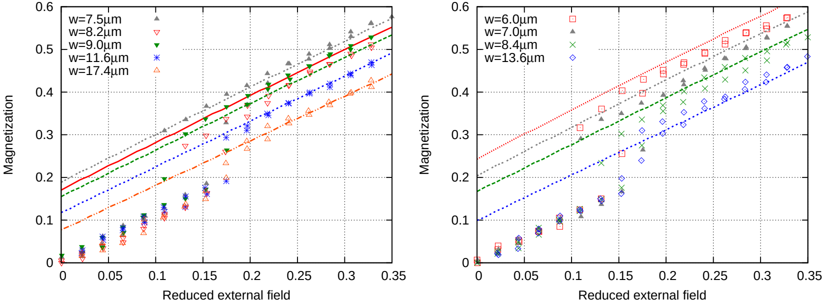
Comparison of theory and experiments

$Co_{40}Fe_{40}B_{20}$ films (lateral width 60μm) with parameters

thickness/nm	102	153	212
$Q/10^{-3}$	1.36	0.93	1.16

$\mu_0 M_s = 1.48$T (measured in a single film of small thickness)
$d = 3.86$nm (from Conca et al., J. Appl. Phys., 2013)

For $2t = 102$nm:



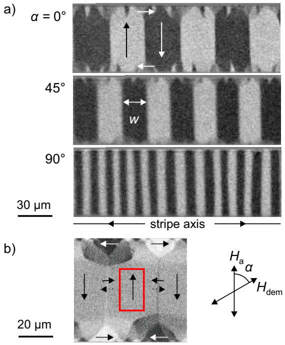
Experiments: C. Hengst

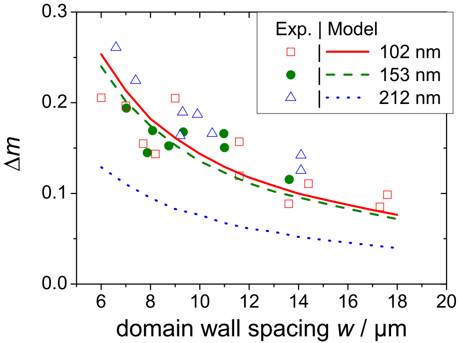
Comparison of theory and experiments

$Co_{40}Fe_{40}B_{20}$ films (lateral width 60μm) with parameters

thickness/nm	102	153	212
$Q/10^{-3}$	1.36	0.93	1.16

$\mu_0 M_s = 1.48$T (measured in a single film of small thickness)
$d = 3.86$nm (from Conca et al., J. Appl. Phys., 2013)

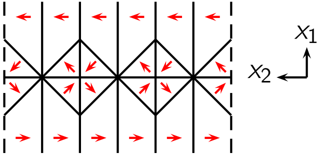




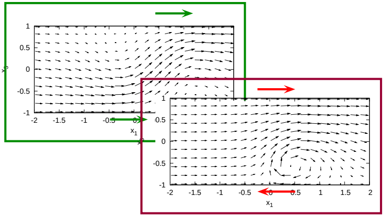
Experiments: C. Hengst

Further questions

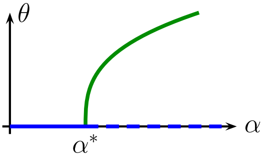
Transversal (in)stability and path to cross-tie wall



Stability of asymmetric walls

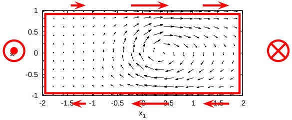


Comparison of critical wall angle $\alpha^* \approx \arccos(1 - \frac{2}{\lambda})$ ($\lambda \approx \frac{t^2}{d^2 \ln \frac{1}{Q}}$) to experiments

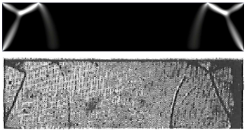


Further questions

Existence of stray-field free walls under degree constraint (energy of div.-free bubbles?)



Thin-film numerics with realistic wall-energy density



D., Esselborn, Ferraz-Leite, Otto

Van den Berg, Vatvani

LLG evolution for unwinding walls: Fast relaxation in core – slow wall motion?