
Experimental Validation in Low-Speed Rarefied Flows: An Overview of Technological Limits and Selected Results

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- **Introduction**
 - Wind Tunnel Measurements
 - Sensitivity in Resonator Systems
 - Boundary Layers on Rotating Disk- Experimental Design
 - Momentum Accommodation Results
 - Conclusions

Transition Length Scales



- Modern era of micro-scale rarefied flow measurements kicked off with discovering unusual pressure drops in micro-channels with gas flow.
- Flows rarefied, with large density changes.
- Knudsen number $Kn = \lambda/D$ could vary over length of system.

Knudsen
Number:

0.001

0.1

1.0

10

Flow
Regime:

Continuum
Flow

Slip Flow

Transitional Flow

Free-
Molecular
Flow

Governing
Equations:

Navier-Stokes
Equations

Navier-Stokes
Equations w/
Slip

Boltzmann Equations with
Collisions

Collision-less
Boltzmann
Equation

Computational
Method:

Conventional CFD

Burnett Solver, Information-
Preservation Method, Direct-
Simulation Monte-Carlo

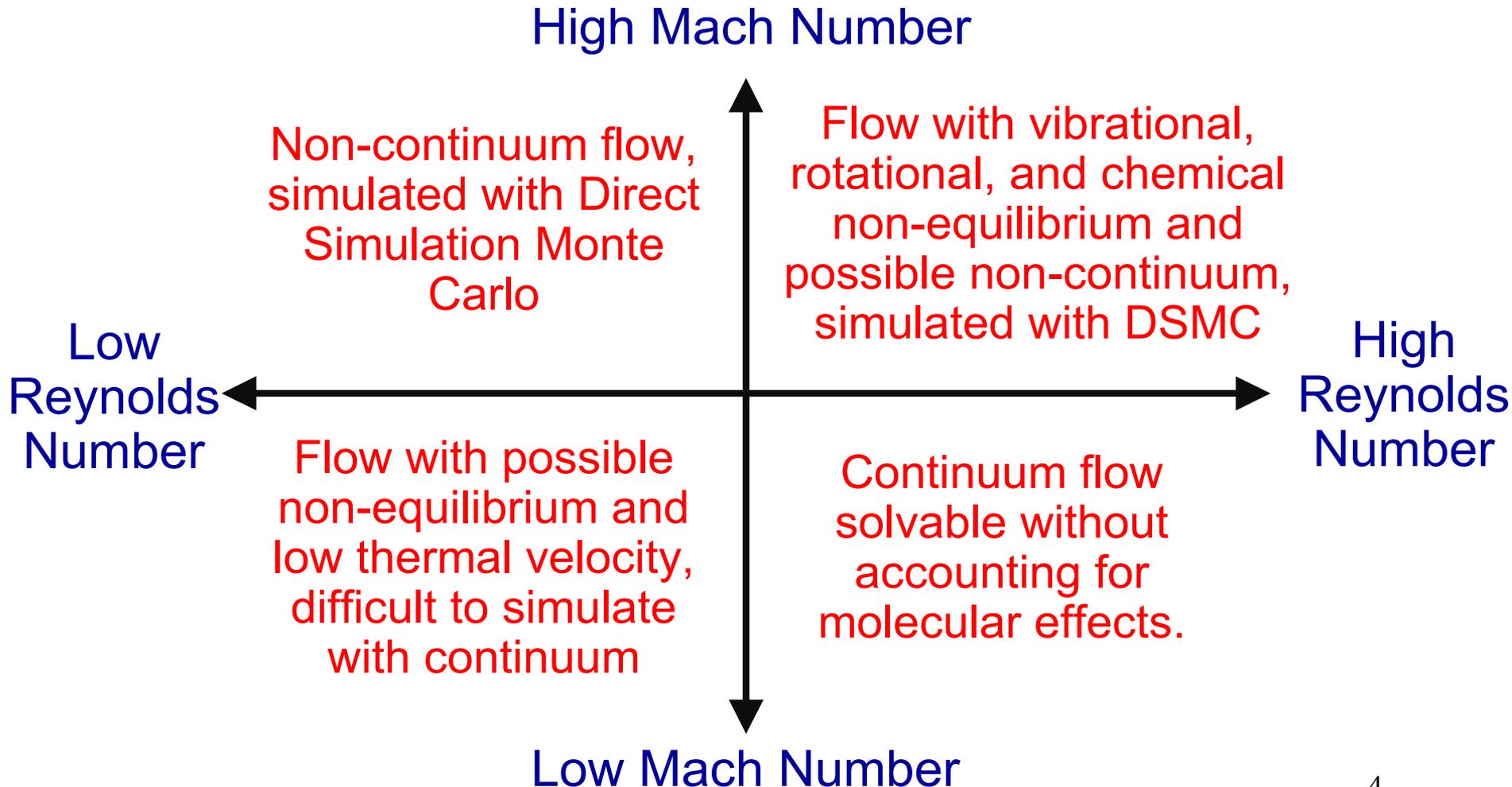
Direct-
Simulation
Monte-Carlo



Transition Length Scales



- Well-known relationship $Kn \approx M/Re$



- Relatively few flows are characterized for continuum breakdown across the whole range from free-molecular to continuum.
- Relatively few effective measurements:
 - Particle Image Velocimetry limited by size of particles, small particle volumes.
 - Measurement of local shear stresses or pressure difficult, but integrated forces sometimes possible.
 - Measurements with more extensive diagnostics often hypersonic, which introduces physics beyond continuum breakdown.

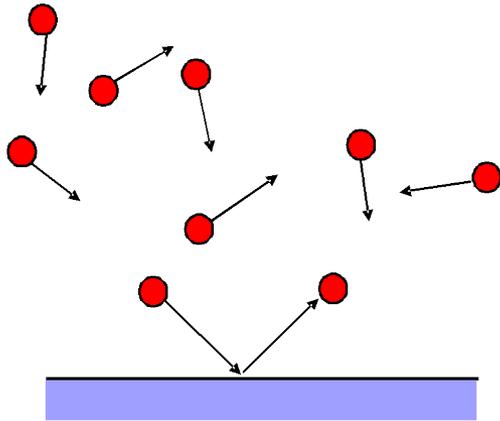
- So why do we need these measurements?

1. Validation of basic physics- until we have experimental results that we can directly compare to, how do we know if our results are good?

2. Need for basic physical parameters, such as wall interactions, to feed into simulations.

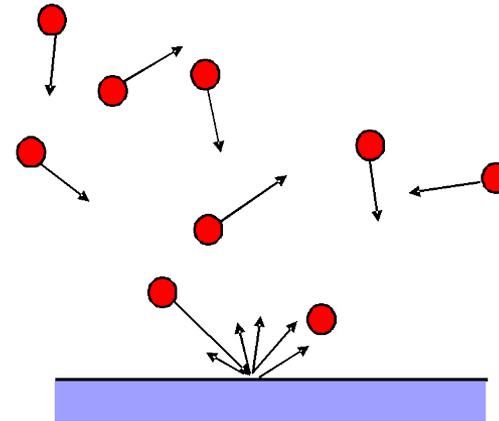
Momentum Accommodation

- Shear stress, heat transfer in rarefied flows depend upon momentum accommodation coefficient σ_t ,



$\sigma_t = 0$ (Specular reflection)

- Gas particle reflected like bouncing ball.



$\sigma_t = 1$ (Diffuse reflection)

- Gas particle re-emitted in random direction with velocity set by wall temp.
- σ_t usually between 0 and 1, but values larger than 1 measured, suggesting back-scattering.
- Similar coefficients for normal momentum, heat transfer.

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Aerodynamic Measurement for Micron-Scale Airfoils



Thesis research- Attempt to measure the lift and drag on flat-plate airfoils in the rarefied flow regime.

- Part 1- Scale laminar boundary layers to determine when slip will occur. Results over-turned accepted wisdom that slip would not matter in a boundary layer, and showed that a 100 micron chord airfoil would have a reduction in drag due to slip.
- Part 2- Design, fabricate, and test a wind-tunnel that could accommodate an airfoil with a span of 1 cm, and allow micro-structure mounting.
- Part 3- Design and fabricated an integrated micro-device/micromachined airfoil.

Facility Requirements

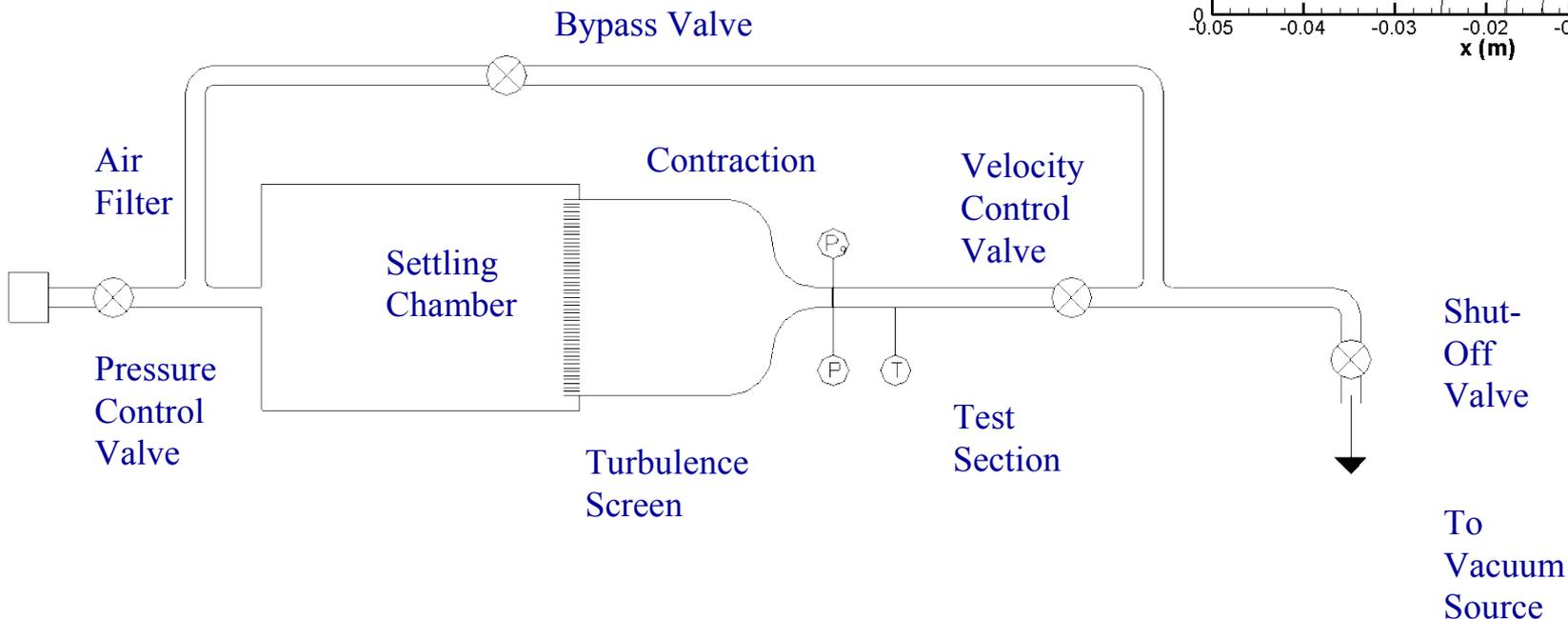
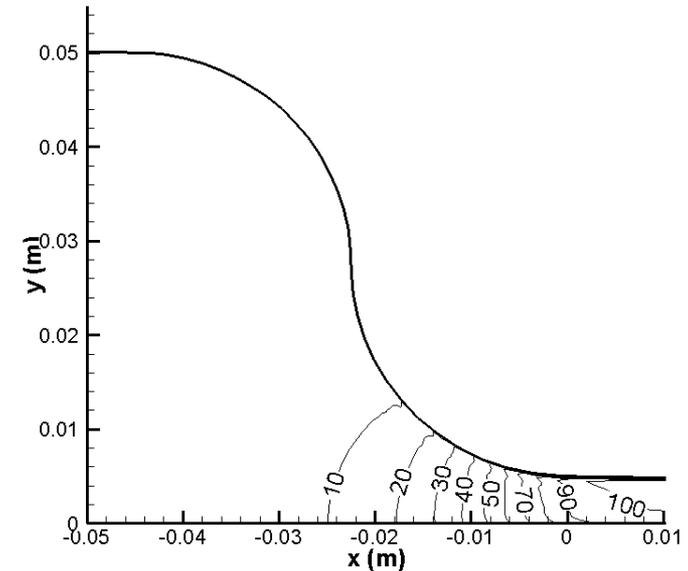


- Simultaneous with sensor design, a special facility built for testing of MEMS scale airfoils:
 - Velocity 10-100 m/s
 - Pressure from 0.1 to 1.0 atmospheres
 - Independent control of Reynolds number and Knudsen number
 - Low turbulence (Less than 0.5 %)
 - Uniform flow across 1 cm test section, with minimal boundary layer

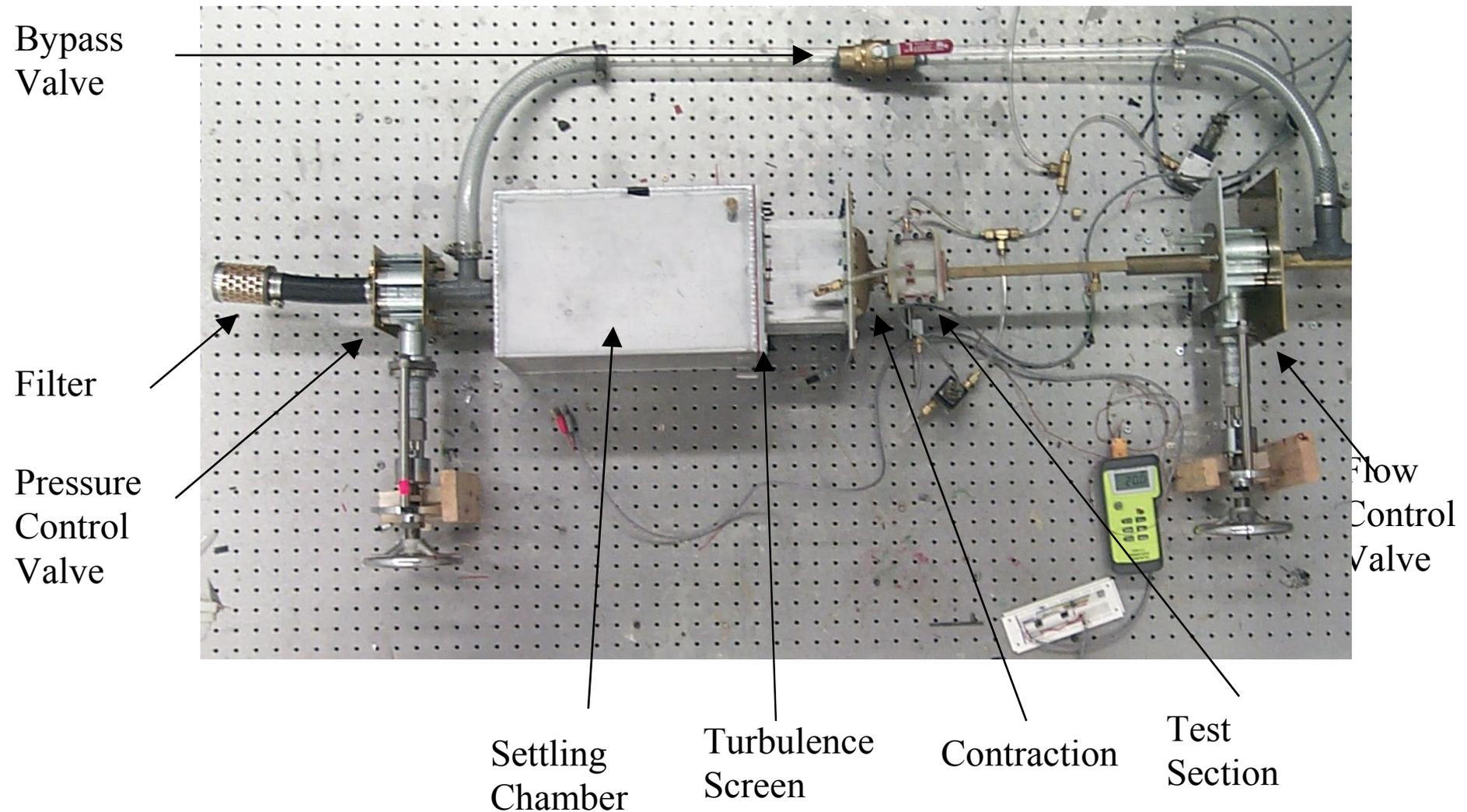
Martin, M. J., Scavazze, K. J., Boyd, I. D., and Bernal, L. P., Design of a Low-Turbulence, Low-Pressure Wind-Tunnel for Micro-Aerodynamics, Journal of Fluids Engineering, Vol. 128(5), pp. 1045-1052, 2006

Facility Configuration

- Configuration selected draw-through wind-tunnel
 - Small size of test section gives relatively large freedom in design, use of a 100-1 contraction main challenge.

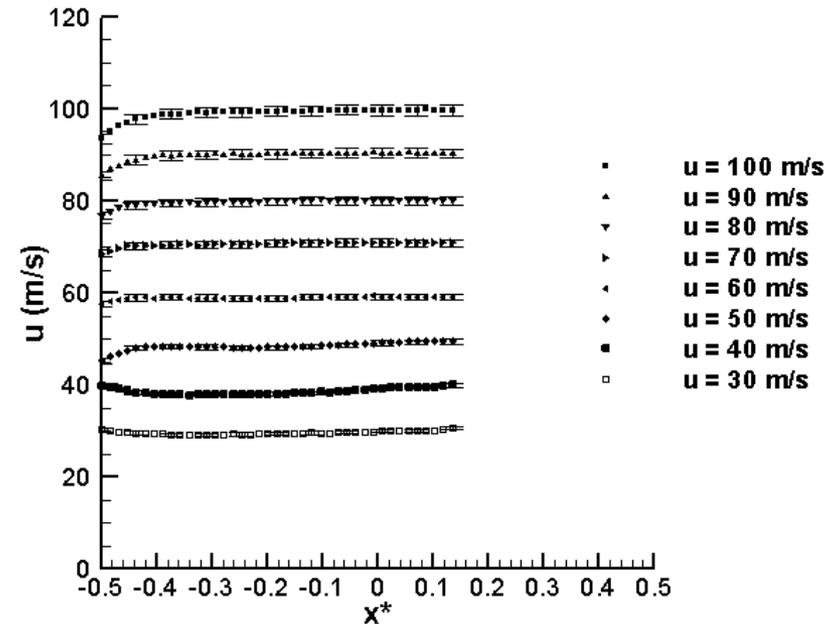
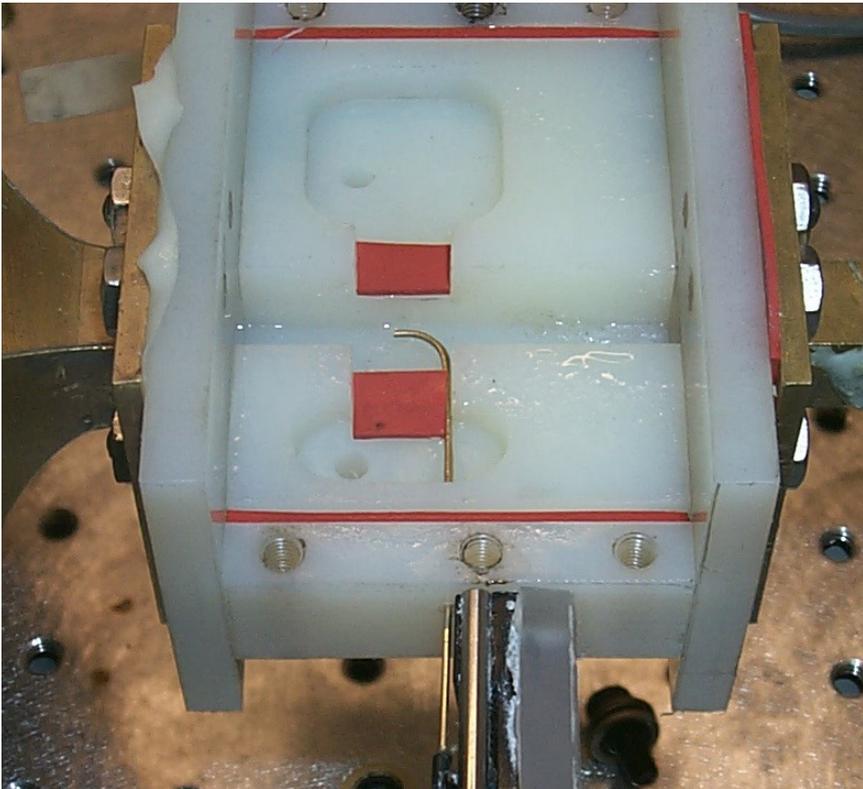


Facility Fabrication



Velocity Measurements

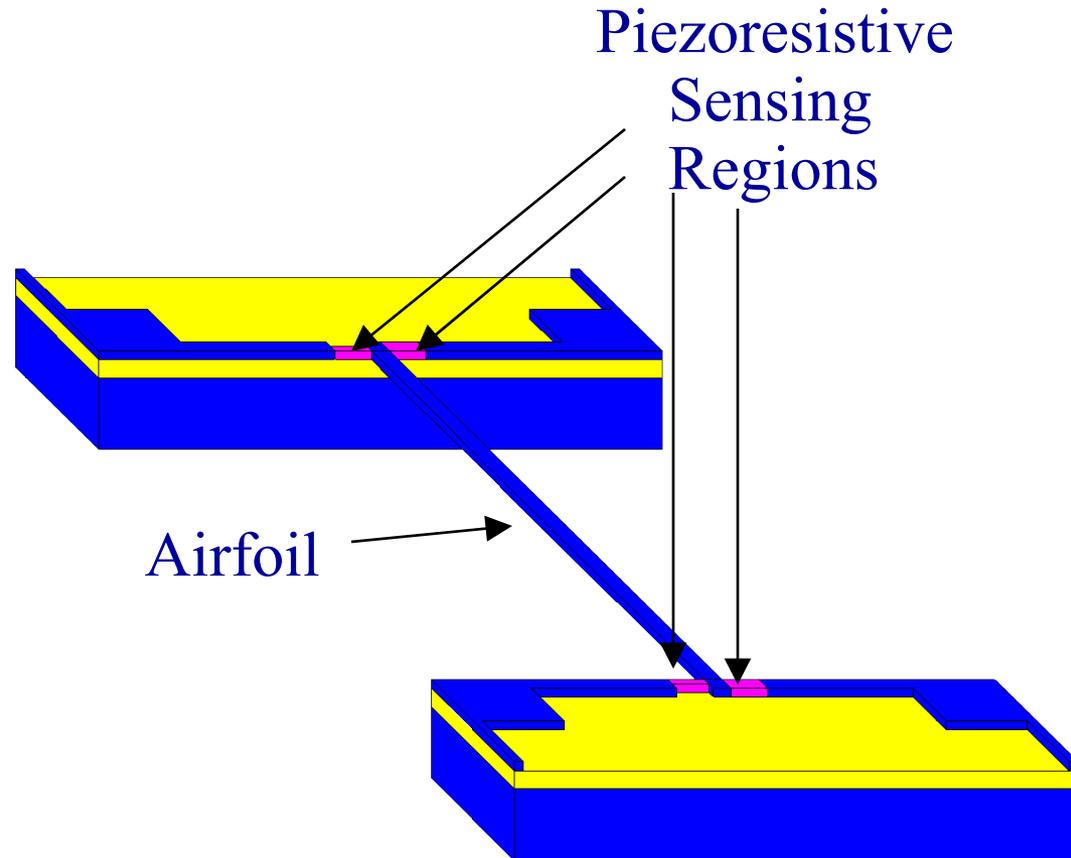
- Velocity across test section measured using impact probe:



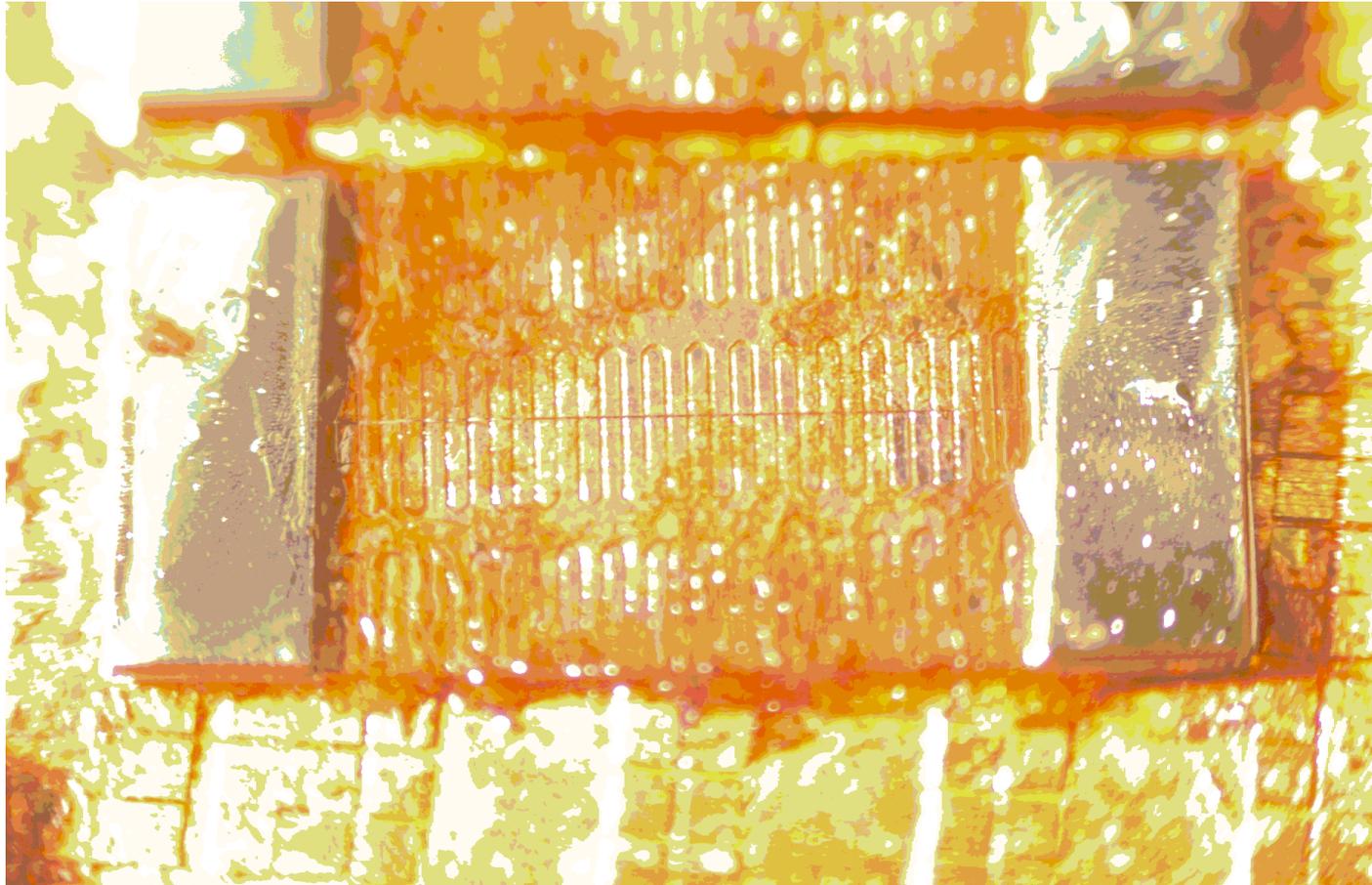
- Additional hot-film measurements show turbulence levels below 0.5 %

Integrated Airfoil/Sensor Design

- Airfoil and piezoresistive region fabricated from SOI (Silicon on Insulator) Wafer.
- Forces on airfoil transmitted to piezoresistive sensing regions.
- Asymmetry of design allows separation of X and Y components of aerodynamic forces
- Electrical connections for a Wheatstone bridge can be incorporated on-chip

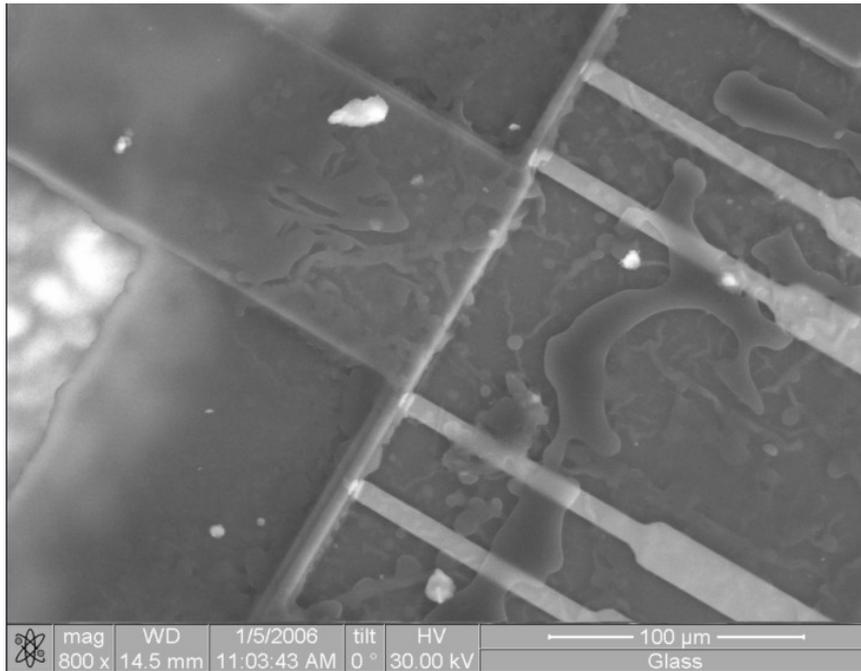


Sensor Fabrication

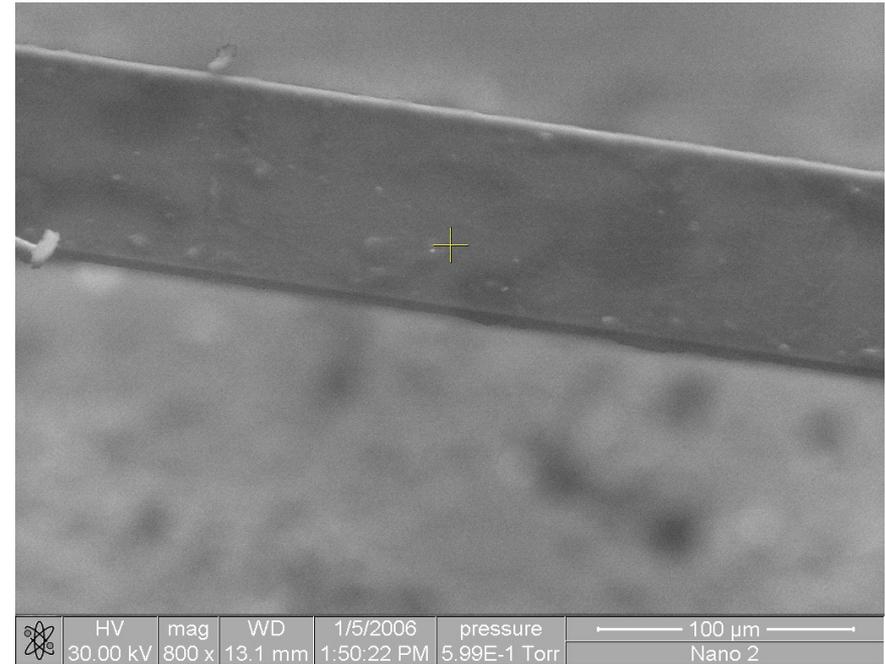


Device before release

- SEM photos show flat-plate airfoil structure:

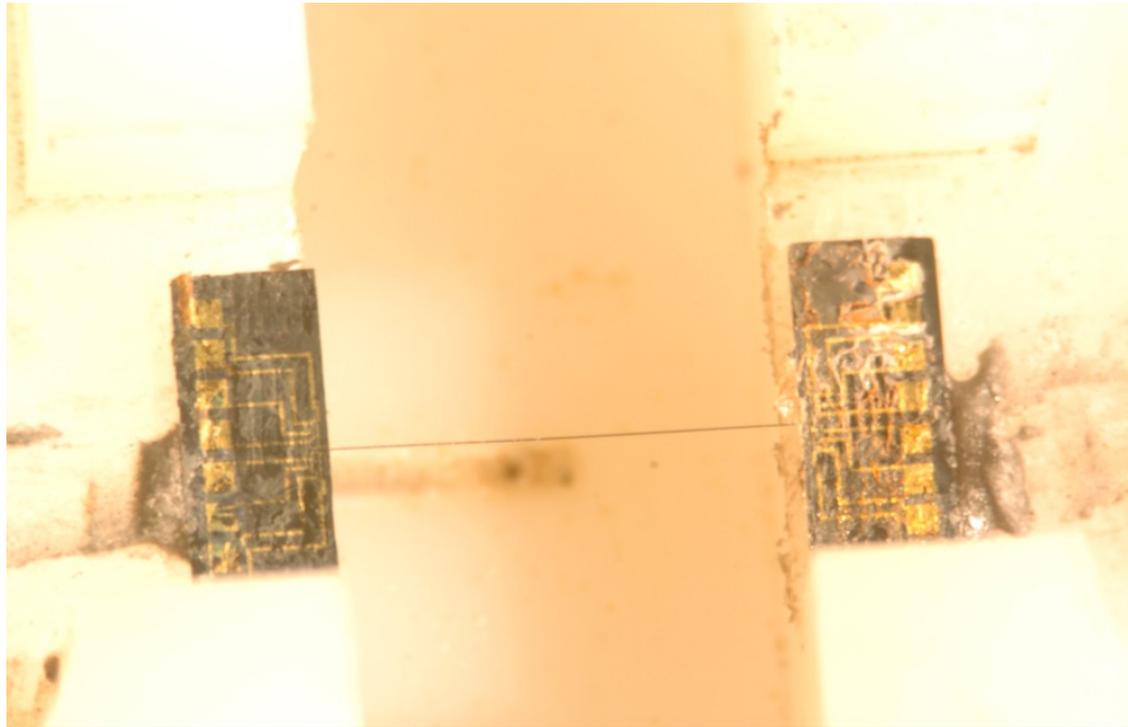


Force sensor and Mounting



Airfoil span

- Airfoils successfully released into wind-tunnel test-section using an acetone bath and mechanical positioning:



So where are the results?

- The device was subject to vortex shedding- a result not predicted by steady CFD.
- All airfoils broke in testing.
- A 2nd generation tunnel might have succeeded, but is this really the best way to get measurements of a rarefied external flow?

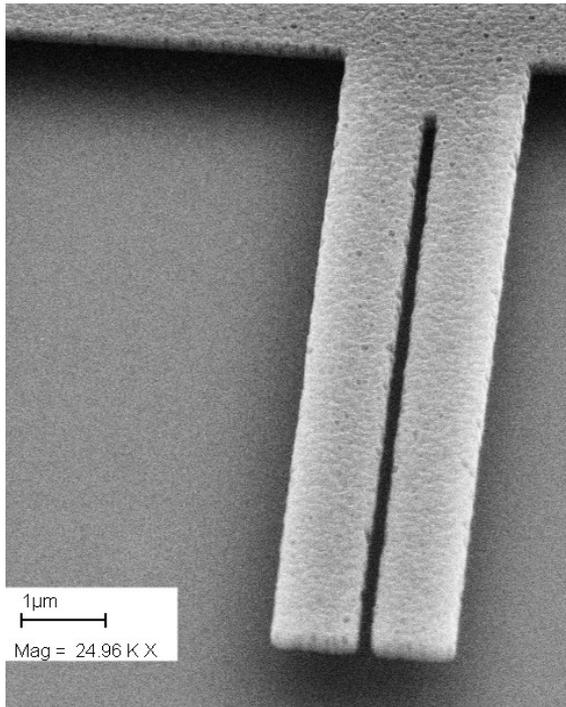
“It takes sixty-five thousand errors before you’re qualified to make a rocket”

- James Michener, in

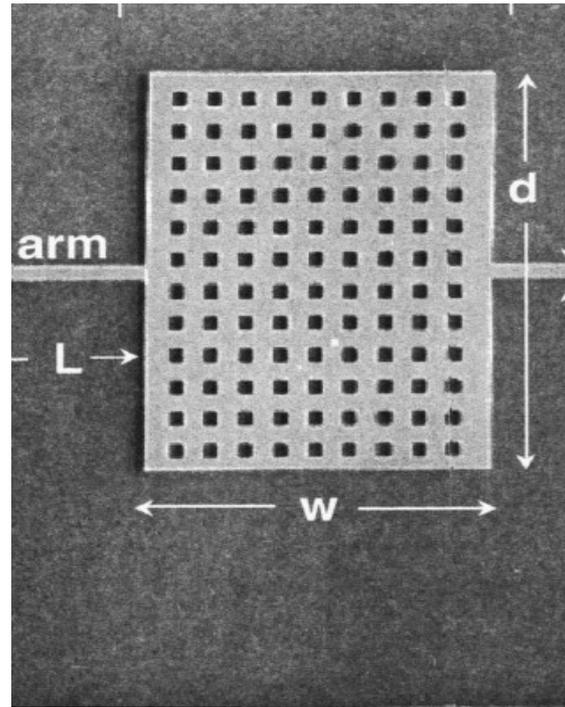
“Space”

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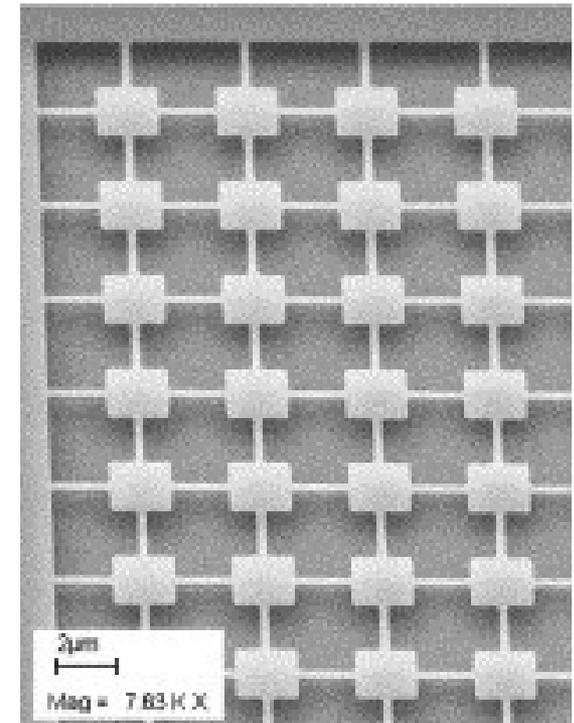
- Variety of Geometries for Micro- and Nano-Scale Resonators



Micro-machined diamond tuning fork resonator



Paddle resonator



Array for signal processing

Damping Mechanisms



• Figure of merit for these systems is the Quality Factor Q , the ratio of the vibrational energy of the system U_i to the loss of energy per cycle U_d : $Q = 2\pi U_i / U_d$

- The loss U_d is a sum of the losses from variety of mechanisms:

$$U_d = U_{structural} + U_{thermoelastic} + \dots + U_{fluid} = U_{int} + U_{fluid}$$

• At any condition other than high vacuum, U_{fluid} is the dominant term, and the quality factor is written as:

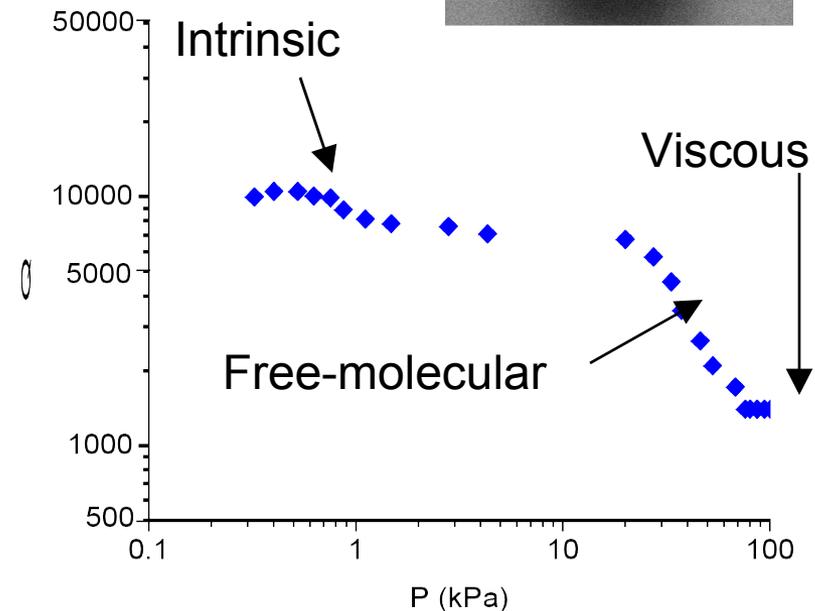
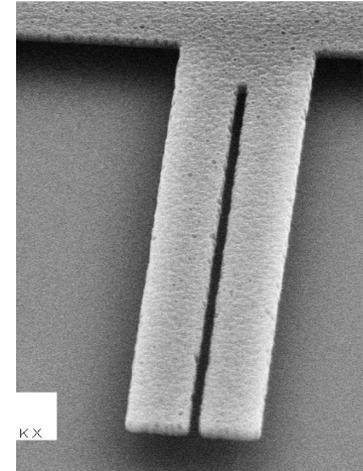
$$\frac{1}{Q} = \frac{U_d}{2\pi U_i} = \frac{U_{int} + U_{fluid}}{2\pi U_i} = \frac{1}{Q_{int}} + \frac{1}{Q_{fluid}}$$

• U_{int} is very easy to measure, but very difficult to compute reliably- but usually much lower than U_{fluid}

Damping Regimes

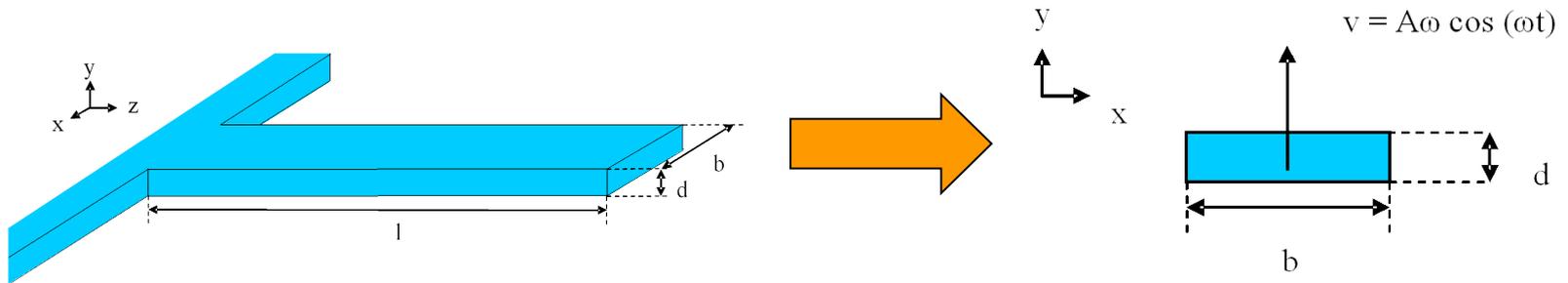
Previous researchers identified 3 damping regimes for a micro-resonator based on pressure:

- Intrinsic damping regime
 - Fluidic losses negligible
- Free-molecular damping regime
 - Gas particles do not collide enough to maintain continuum
- Viscous damping regime
 - Classical fluid mechanics



Turning Drag into a Quality Factor (1)

- Move from 3-D geometry to 2-D cross-section



3-D Cantilever Geometry

2-D Cantilever Geometry

- Give the system a motion of amplitude A and frequency ω :

$$y(t) = A \sin(\omega t), v(t) = A\omega \cos(\omega t)$$

- The vibrational energy will be equal to the peak kinetic energy:

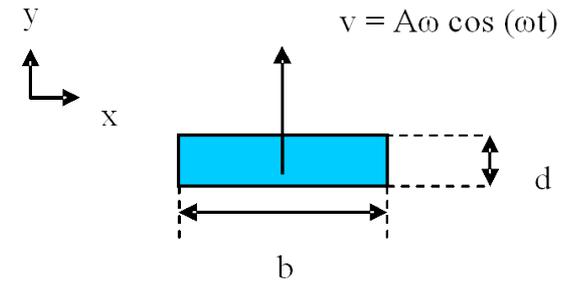
$$U_i = \rho_s b d (A\omega)^2 / 2$$

Turning Drag into a Quality Factor (2)



- Need to make a few estimates about the cantilever motion

$$v(t) = A\omega \cos(\omega t)$$



2-D Cantilever Geometry

- Amplitude of displacement of MEMS/NEMS typically 0.1 -100 nm

- For a lightly damped system $\omega \approx \omega_n$. Obtain ω from beam theory:

$$\omega_n = \left(k_n/l\right)^2 \sqrt{EI/M}$$

- E is the elastic modulus, I is the moment of inertia, M is the mass per length, k_n is the mode constant (1.875 for cantilever in 1 mode, 4.730 for bridge)

- Result- max velocity usually well below 1 m/s.

- Free molecular flow drag and heat transfer are well-studied.
- Shear stress τ , pressure P , and energy flux $\Delta\Phi$ can be found using kinetic theory:

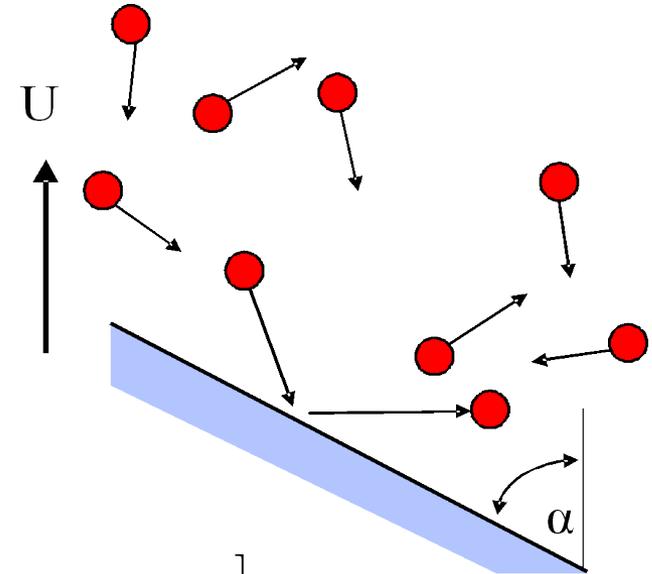
$$\tau = \sigma_t P_i s \left[\frac{\exp(-s_3^2)}{\sqrt{\pi}} + s_3 [1 + \operatorname{erf}(s_3)] \right]$$

$$P = P_i \left[\left(\frac{2 - \sigma_n}{\sqrt{\pi}} s_3 + \frac{\sigma_n}{2} \sqrt{\frac{T_w}{T_i}} \right) \cdot \left(\exp(-s_3^2) + \sqrt{\pi} s_3 [1 + \operatorname{erf}(s_3)] \right) + \frac{2 - \sigma_n}{2} [1 + \operatorname{erf}(s_3)] \right]$$

$$\Delta\Phi = \sigma_E P_i v_i \left[\exp(-s_3^2) \left(s^2 + \left(\frac{\gamma + 1}{2(\gamma - 1)} \right) \left(1 - \frac{T_w}{T_i} \right) \right) + \sqrt{\pi} s_3 [1 + \operatorname{erf}(s_3)] \left(s^2 + \left(\frac{\gamma}{\gamma - 1} \right) + \left(\frac{\gamma + 1}{2(\gamma - 1)} \right) \left(\frac{T_w}{T_i} \right) \right) \right]$$

$$s = \sqrt{\frac{m}{2k_b T_i}} u$$

$$s_3 = \sin(\alpha) \cdot s$$



- σ_t and σ_n are tangential and normal momentum accommodation coefficients.
- σ_E is the thermal accommodation coefficient
- P_i and T_i are ambient pressure and temperature.

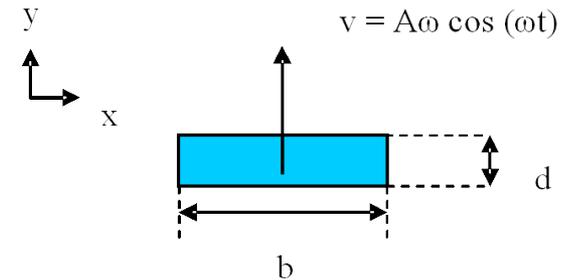
- T_w is the wall temperature
- m is mass of a molecule
- k_b is the Boltzmann constant
- v_i is the average particle velocity
- γ is the specific heat ratio

Finding Free-Molecular Drag

- Finding the drag on the cantilever :

$$F_D = b(P_{top} - P_{bottom}) + 2d\tau_{sidewall}$$

- Use $\alpha = 0^\circ$ on top, 90° on sidewall, 180° on bottom to obtain



2-D Cantilever Geometry

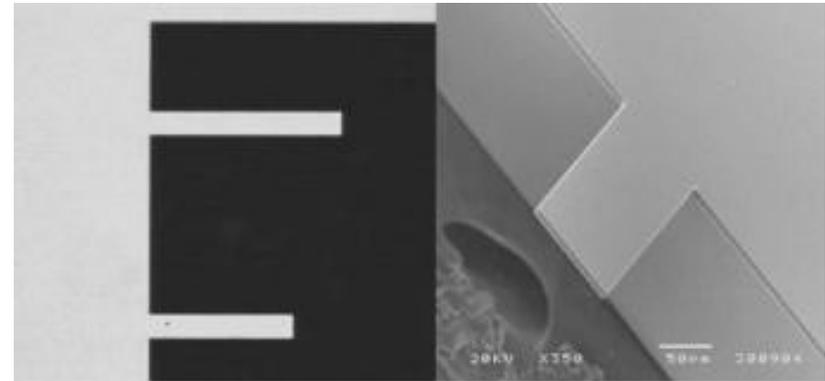
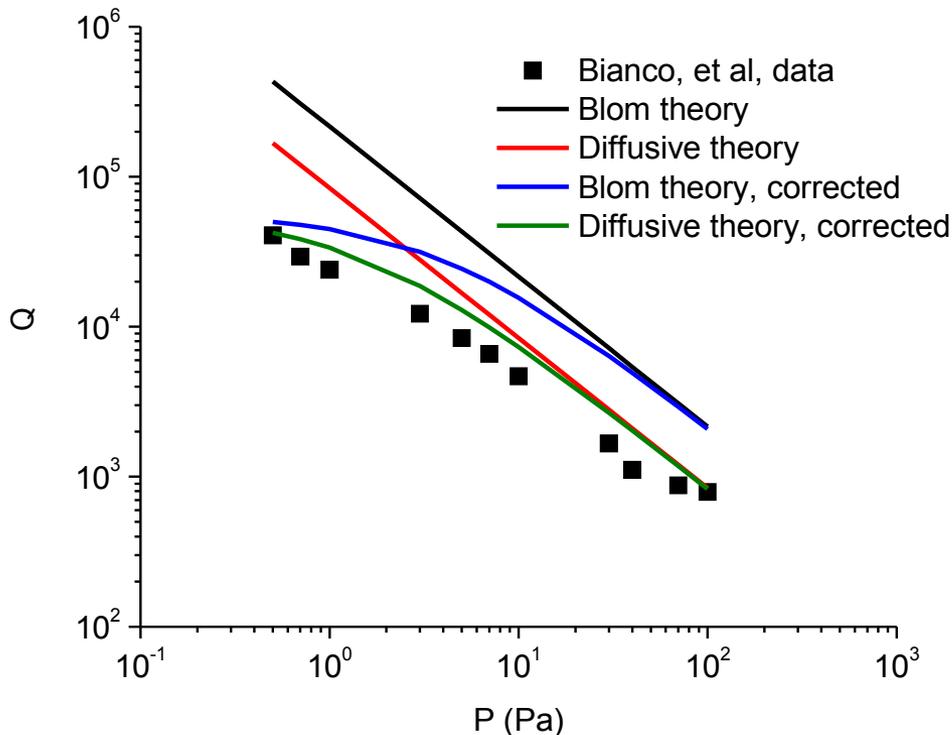
$$F_D = bP_i \left[(2 - \sigma_n) \cdot \left(\frac{2s}{\sqrt{\pi}} \exp(-s^2) + (2s^2 + 1) \operatorname{erf}(s) \right) + \sigma_n \pi^{1/2} s \sqrt{\frac{T_w}{T_i}} + 2 \frac{d}{b} \frac{\sigma_t s}{\pi^{1/2}} \right]$$

- Assume $s \ll 1$, which is true at low Mach numbers, and the equation linearizes to:

$$F_D = \frac{bP_i u}{c} \left[(2 - \sigma_n) \left(\frac{2}{\sqrt{\pi}} + 1 \right) + \sigma_n \pi^{1/2} \sqrt{\frac{T_w}{T_i}} + 2 \frac{d}{b} \frac{\sigma_t}{\pi^{1/2}} \right]$$

Free-Molecular Results

- Comparison with experimental data from micro-cantilevers: 600 μm long, 100 μm wide, 5 μm thick silicon cantilever at $\omega = 1.15 \times 10^5 \text{ s}^{-1}$



Bianco, et al, *J. of Vacuum Science & Technology B*, **24** 1803, 2006

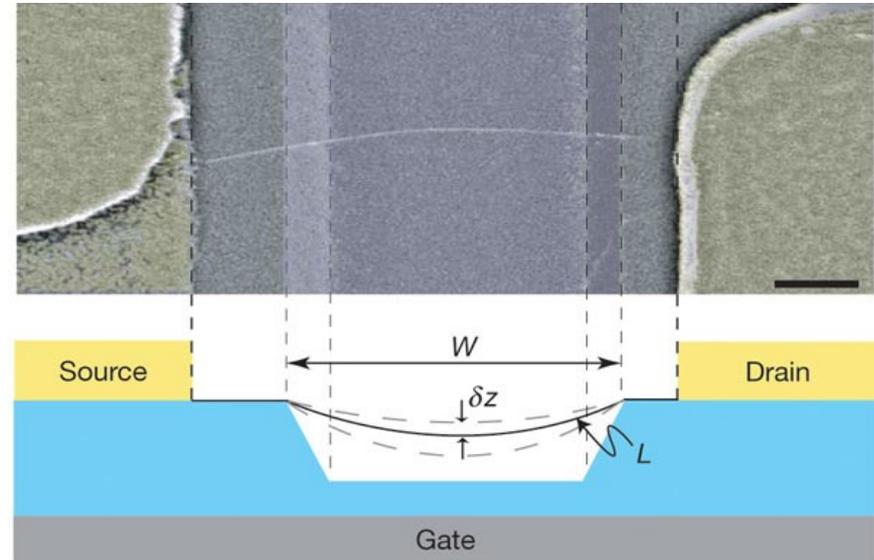
- Similar results for 800 μm long, 100 μm wide, 5 μm thick silicon cantilever at $\omega = 6.47 \times 10^5 \text{ s}^{-1}$ Martin et al *JMEMS*, 2008.

Carbon Nanotube Resonator (1)

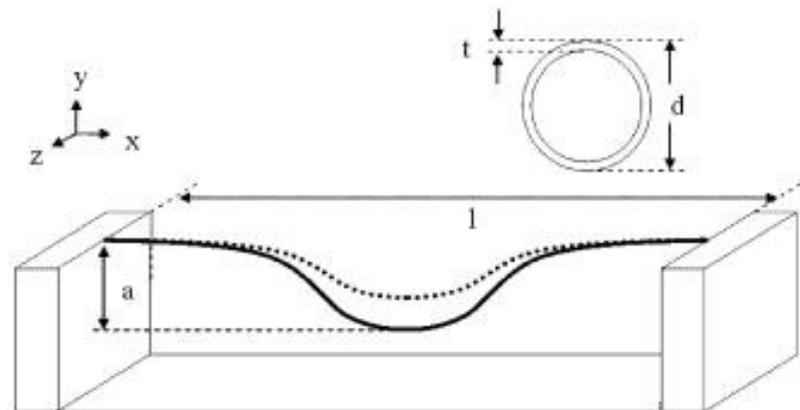
- How far can these models be extended ?
- Single-wall carbon nanotube resonators have been fabricated and tested
- Small (nm diameter) means free molecular flow even at atmospheric pressure

$$Kn = \frac{\lambda}{D} \approx \frac{50 \text{ nm}}{1.3 \text{ nm}} \approx 11$$

- Continuum mechanics often adequate to model nanotube



Sazonova, et al, *Nature* **431** 284, 2004



Carbon Nanotube Resonator (2)

- Continuum mechanics gives I:

$$I = \pi \left(d^4 - (d - 2t)^4 \right) / 64$$

- Free-molecular flow theory give the drag on a cylinder

–Maslach and Schaaf, Physics of Fluids 6 315, 1963

$$F_D = d \frac{\pi^{1/2} \rho u^2}{2 \cdot s} \left[\exp\left(-\frac{s^2}{2}\right) \left(\left(s^2 + \frac{3}{2}\right) I_0\left(\frac{s^2}{2}\right) + \left(s^2 + \frac{1}{2}\right) I_1\left(\frac{s^2}{2}\right) \right) + \frac{\pi}{4} \sqrt{\frac{T_w}{T_o}} \right]$$

- Which linearizes to:

$$F_D = u \left(\frac{Pd}{c} \right) \left[1.5\pi^{1/2} + \pi^{3/2} \sqrt{T_w/T_o} \right]$$

- Putting this all together gives Q:

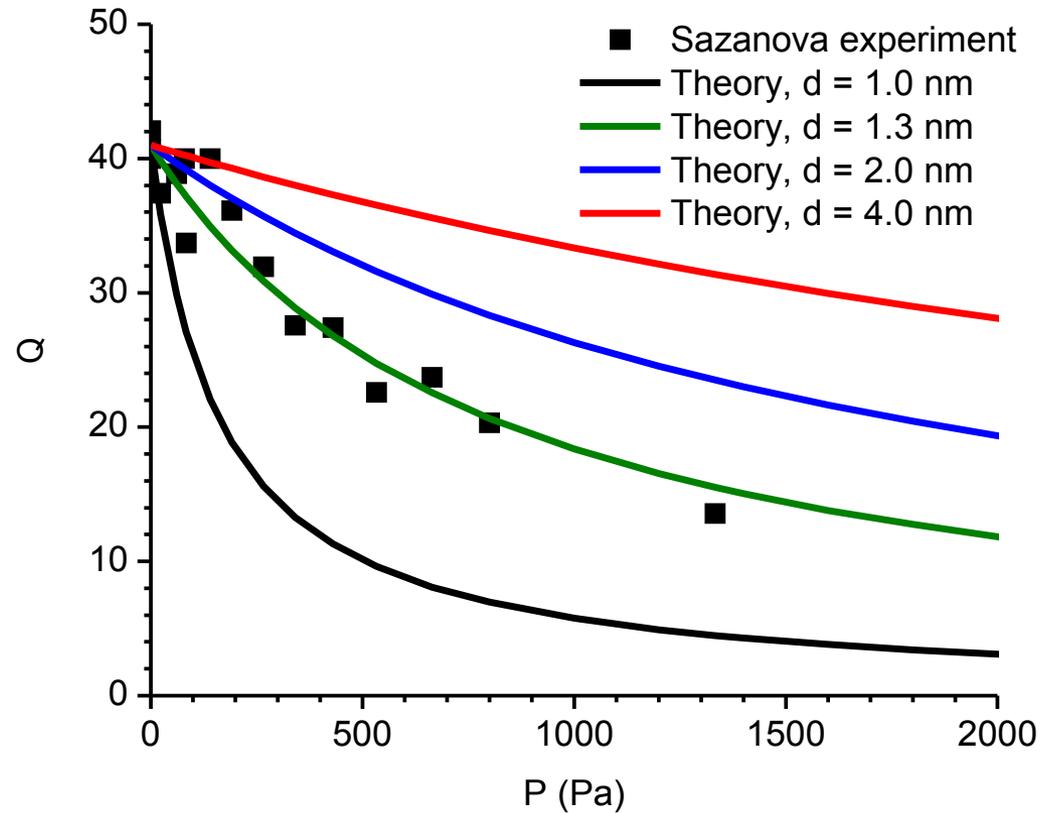
$$Q_f = \frac{\sqrt{\pi/2}}{\left(1.5 + \sqrt{T_w/T_o}\right)} k_n^2 \frac{c}{P} \frac{d}{l^2} \sqrt{E\rho} \sqrt{2d} t \sqrt{1 - 3\left(\frac{t}{d}\right) + \left(\frac{t}{d}\right)^2 - 2\left(\frac{t}{d}\right)^3}$$

Carbon Nanotube Resonator (3)

- Comparison to experimental data complicated by uncertainty in diameter of nanotube (between 1 and 4 nm)

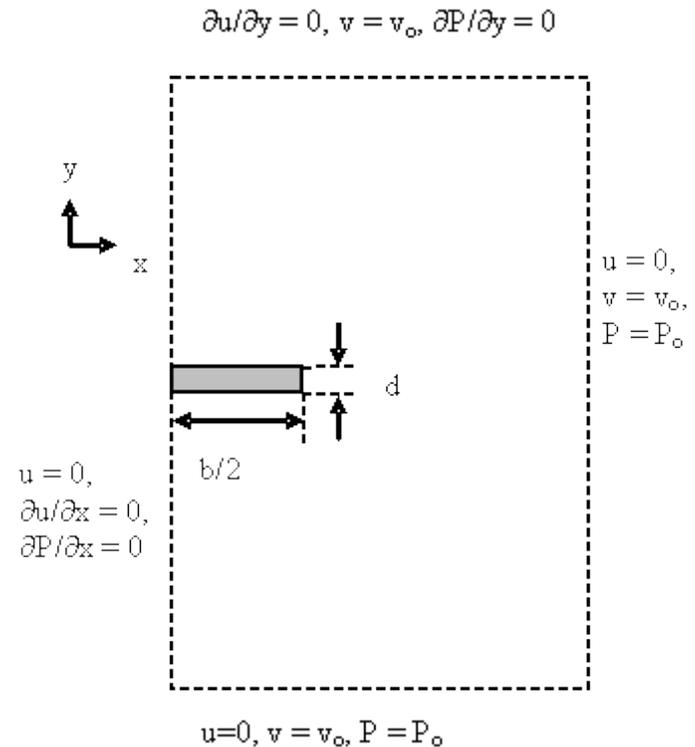
- Diameter of 1.3 nm fits data very well

- Relatively large intrinsic loss



- Martin and Houston, *APL* **91** 103016, 2007

- Flow around half-cantilever simulated using Marker-and-Cell (MAC) viscous flow solver.
- Exterior boundary condition set to either steady velocity (Quasi-steady method) or changed based on time.
- Wall slip condition used to incorporate non-equilibrium effects.
- Integrate force over entire cycle to get work:



Computational
Geometry

$$U_d = \int_0^{2\pi/\omega} F_d(t) \cdot v(t) dt$$

Comparison with Experimental Results



- Experimental data for 2 sets of cantilevers used
(Bianco, et al, *J. of Vacuum Science & Technology B*, **24** 1803, 2006)
 - 200 μm long, 40 μm wide, 5 μm thick silicon cantilever vibrating at $\omega = 1.04 \times 10^5 \text{ s}^{-1}$
 - 600 μm long, 100 μm wide, 5 μm thick silicon cantilever vibrating at $\omega = 1.15 \times 10^5 \text{ s}^{-1}$
- Computational methods used:
 - Unsteady NS, No-slip and slip
 - Quasi-steady NS, No-slip and slip
 - Vibrating sphere model
 - Modified cylinder model

Comparison with Experimental Results



Computational methods compared:

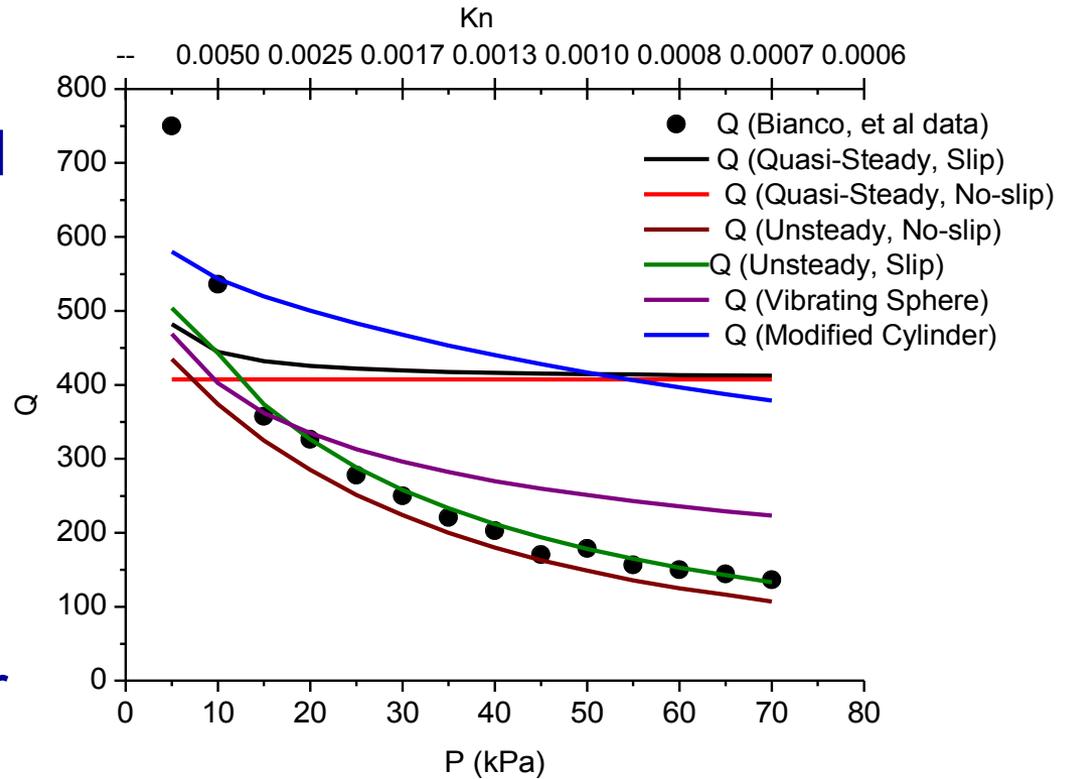
-Unsteady NS, No-slip and slip-

-Quasi-steady NS, No-slip and slip-

-Vibrating sphere model

-Modified cylinder model

Similar results obtained for 200 μm long, 40 μm wide, 5 μm thick cantilever at $\omega = 1.104 \times 10^{-5}$ s



600 μm long, 100 μm wide, 5 μm thick cantilever at $\omega = 1.15 \times 10^{-5}$ s

- Can we use this to measure accommodation coefficients? Probably not:
 - Large variations in material properties from sample to sample.
 - Fabrication of cantilevers from a particular material may not be possible.
 - Measurements error of Q large
- Can we use this method to validate moment methods?
 - Bianco et al have results in a transitional flow regime.
 - Modeling approach doesn't care if your unsteady solver is based on Navier-Stokes or Moment Closure.
 - Probably yes.

Can we get useful information on thermal accommodation from the mechanical response of micro-structures?

- A 3-dimensional doubly clamped bridge is simulated at the micro- and nano-scales to investigate heat transfer effects on the mechanical response of the system.

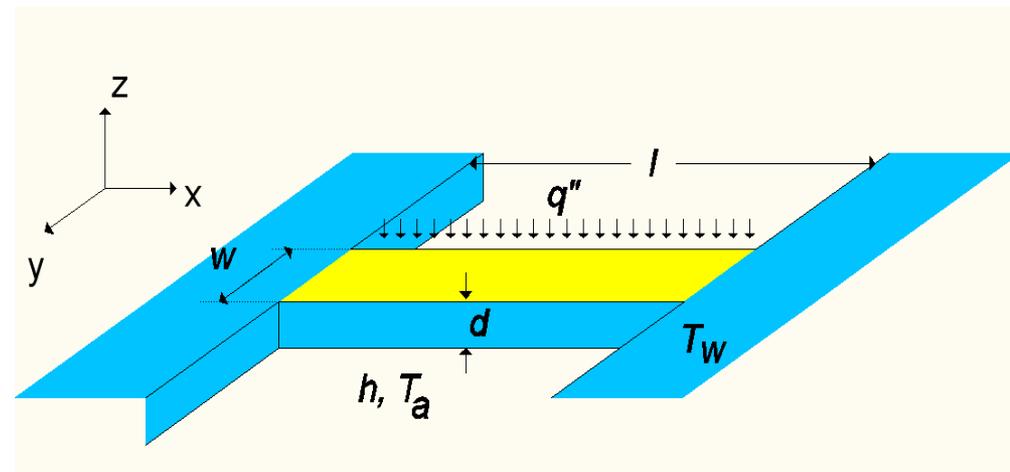
- Boundary Conditions

- Constant T at walls
- Convection at sides (air)
- Constant q'' at the top.

- Governing equation:

- Heat transfer equation
- Thermal stress calculation
- Structural equations.

Geometry



Formulation



- Conduction equation:

$$k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = 0$$

- Thermal stress at each node:

$$\sigma_{Th} = \alpha E \left(T_a - T(x, y, z) \right)$$

- Bending Moment at each plane

$$M(x) = \int_{Area} \sigma_{Th} dydz$$

- Deflection along the length of the beam:

$$\frac{d^2v(x)}{dx^2} = - \frac{M(x)}{EI}$$

Finite Difference structural solver is used to calculate the deflection along the length of the beam.

- σ_{Th} : Thermal stress
- α : Expansion coefficient
- E : Modulus of elasticity
- M : Bending moment.

- T_a : Ambient temp.
- v : Deflection
- I : Moment of inertia

Models for heat transfer to ambient gas



- The energy transfer model from free-molecular flows is valid at high Knudsen numbers:

$$\Delta \Phi = \sigma_E P_i v_i \left[\exp(-s_3^2) \left(s^2 + \left(\frac{\gamma + 1}{2(\gamma - 1)} \right) \left(1 - \frac{T_w}{T_i} \right) \right) + \sqrt{\pi} s_3 [1 + \operatorname{erf}(s_3)] \left(s^2 + \left(\frac{\gamma}{\gamma - 1} \right) + \left(\frac{\gamma + 1}{2(\gamma - 1)} \right) \left(\frac{T_w}{T_i} \right) \right) \right]$$

- At low velocities, linearizing gives a result for heat transfer coefficient h that is independent of velocity:

$$h = \sigma_E \cdot \left(\frac{\gamma + 1}{\gamma - 1} \right) \cdot P \cdot \sqrt{\frac{k_b}{8 \cdot \pi \cdot T_{gas} \cdot m_{gas}}}$$

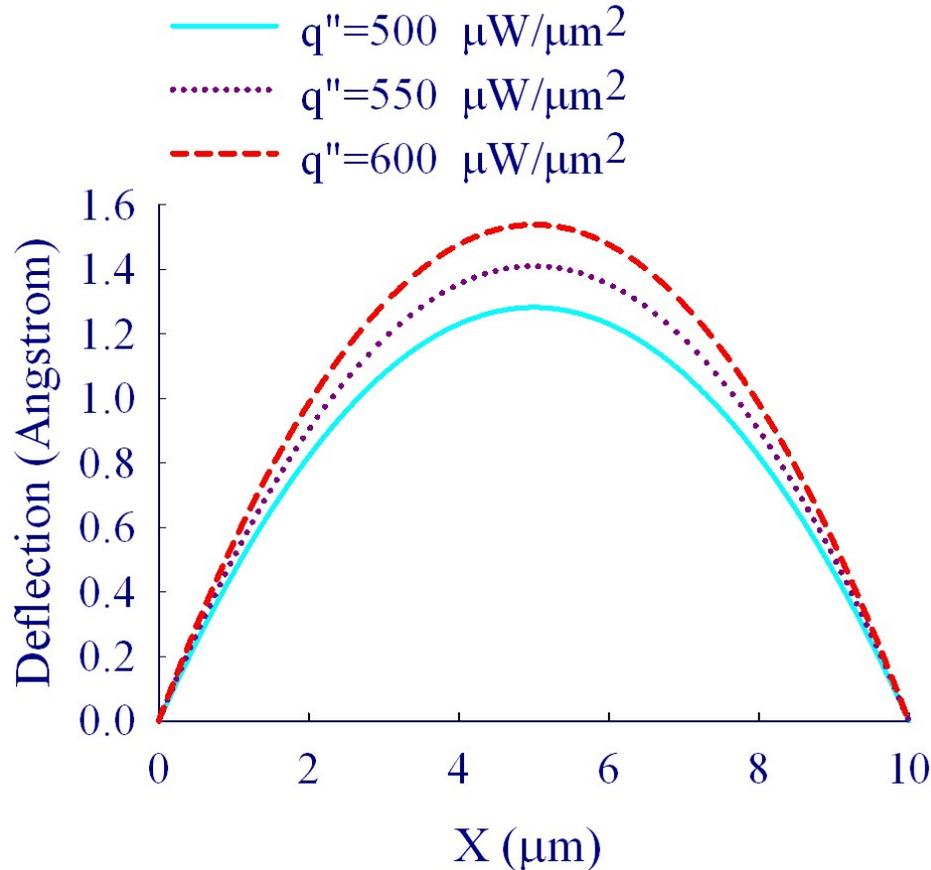
- In the continuum regime, an approximation can be made based on conduction into an infinite medium:

$$h = 0.932 \cdot k_{gas} \cdot \sqrt{\frac{2 \cdot \pi}{w \cdot l}}$$

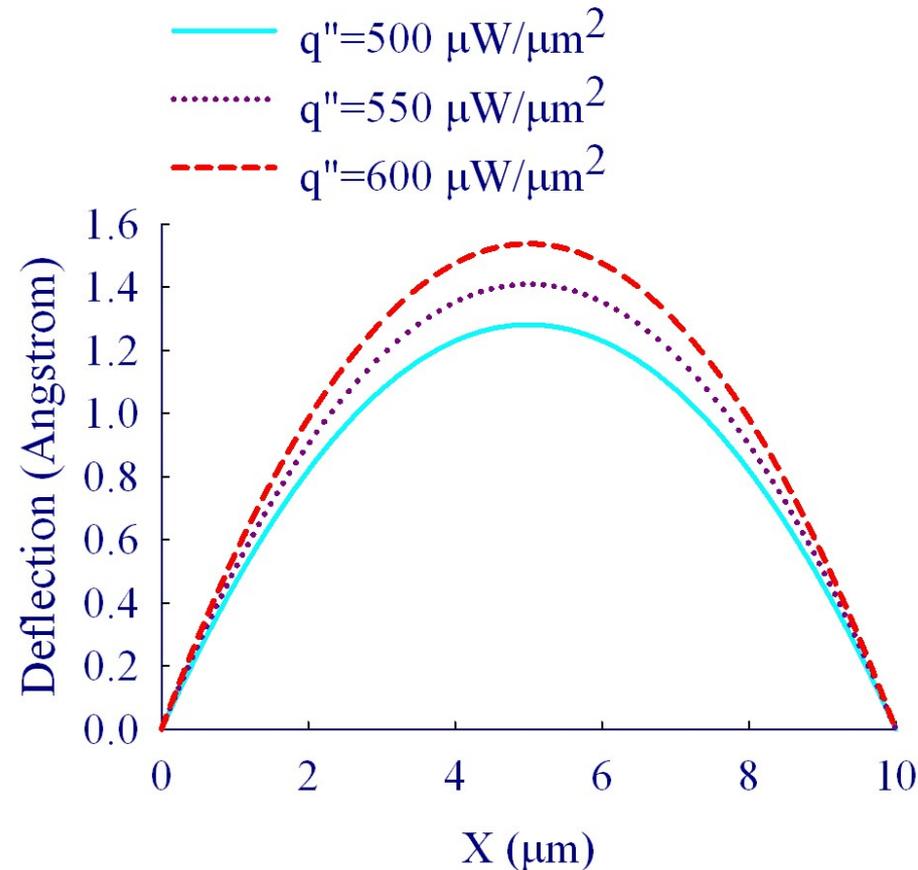
Computational results



Results for 1 μm wide, 10 μm long by 300 nm thick bridge in air.



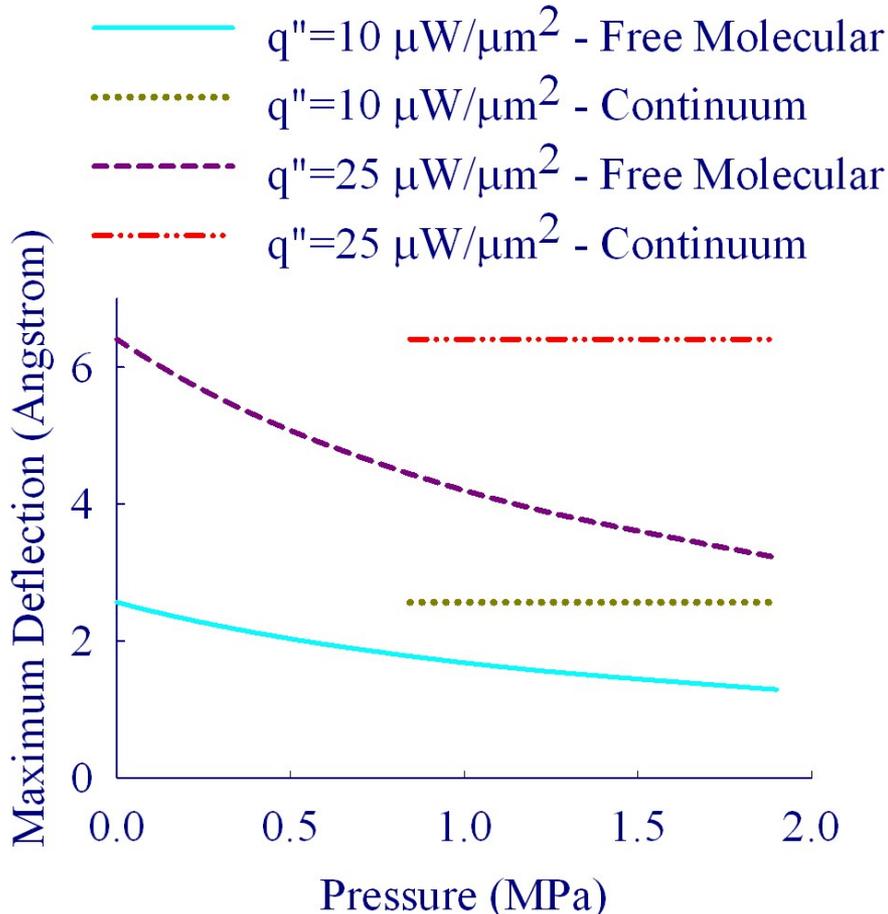
Silicon-Free molecular-Bi=3.2e-8- P=0.1Pa



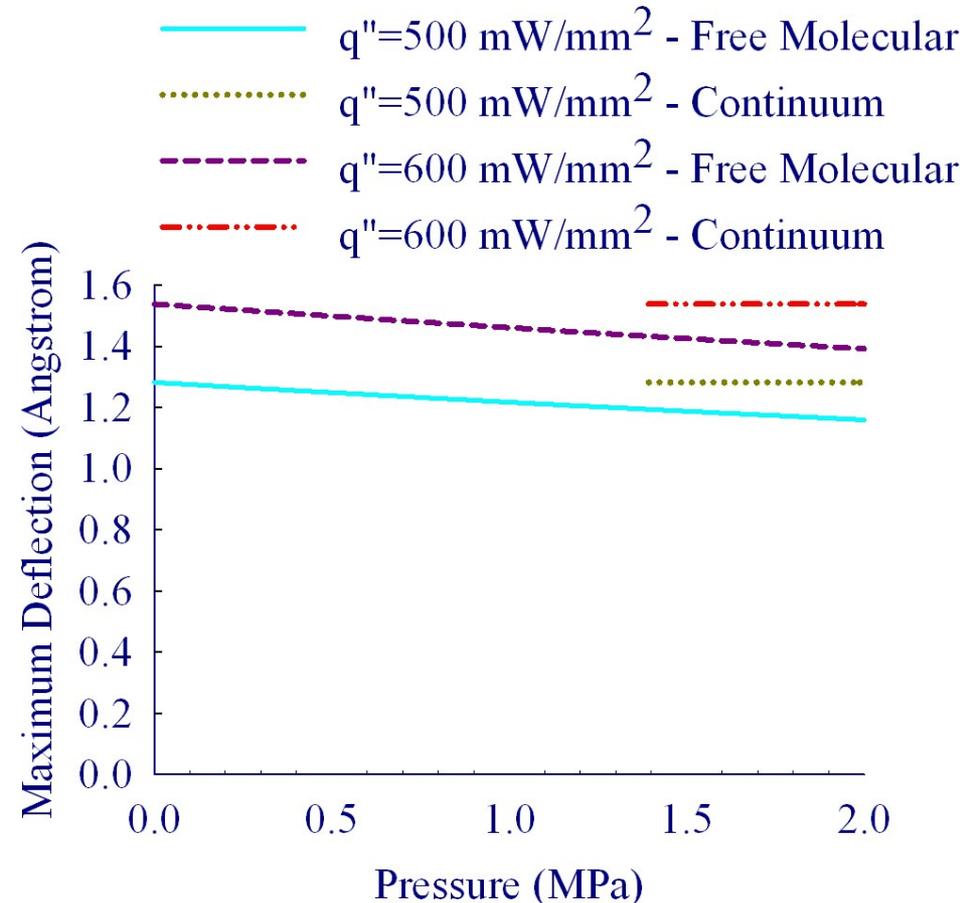
Silicon-Continuum-Bi=1.5e-3

- The displacement behavior is parabolic and the maximum deflection occurs at the center.

Computational results



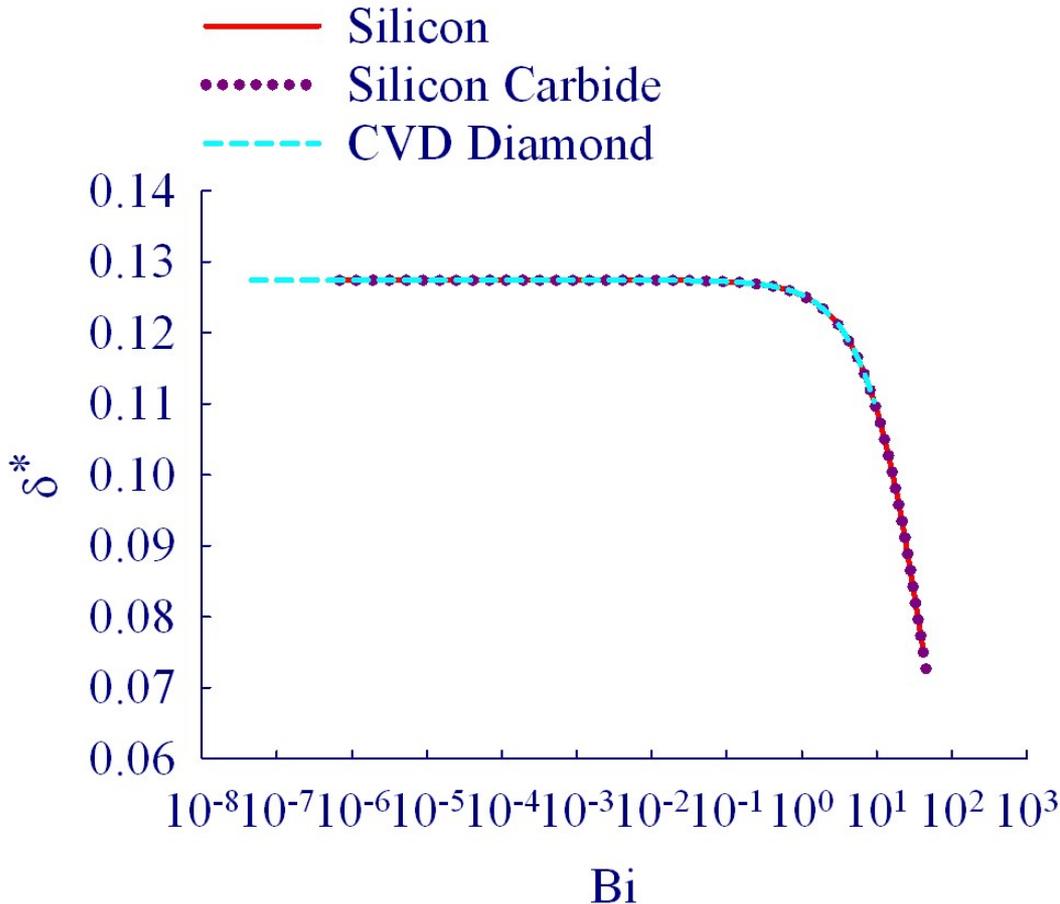
Maximum deflection variation by pressure for Silicon Micro-scale



Maximum deflection variation by pressure for Silicon Nano-Scale

- Maximum deflection increases as the heat load increases.

Computational results



- Use a non-dimensional displacement δ based on heat addition.

$$\delta^* = \frac{\delta k}{q'' \alpha l^2}$$

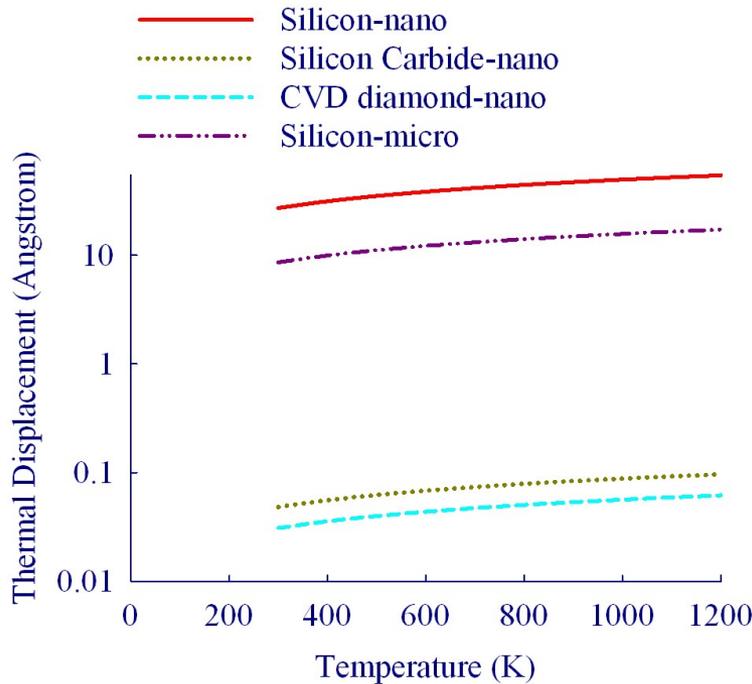
- We compare this result to the Biot number:

$$Bi_l = hl/k$$

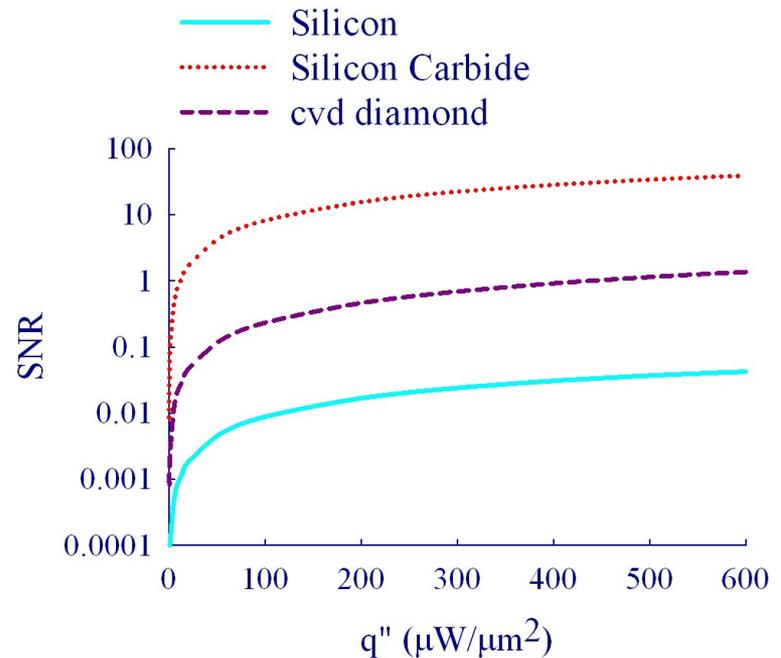
- The results collapse onto one curve when non-dimensionalized.

- Displacement from thermal stresses must be compared to statistical mechanics effects- thermal noise.

- Thermal displacement: $\delta_{Th} = \sqrt{k_b T / k_s}, k_s = 192EI / l^3$
- Signal to noise ratio: $SNR = \delta / \delta_{Th}$



Thermal displacement versus temperature



Signal-to-noise ratio versus heating

-
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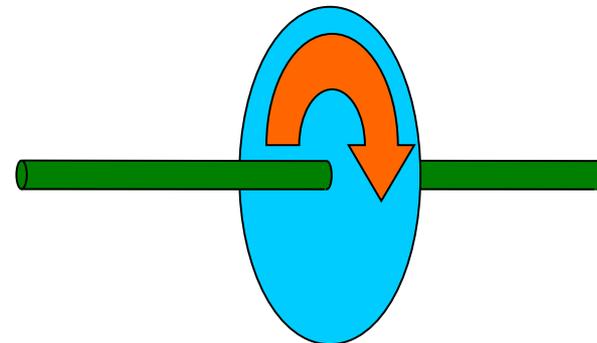
Experimental work- characterizing continuum breakdown

- Can we use a disc-based system to characterize low-speed boundary layer flows?
- Advantages:
 - Mechanical simplicity means less new instrumentation needs to be created.
 - If we get down to free molecular flow we can measure tangential momentum accommodation coefficients.

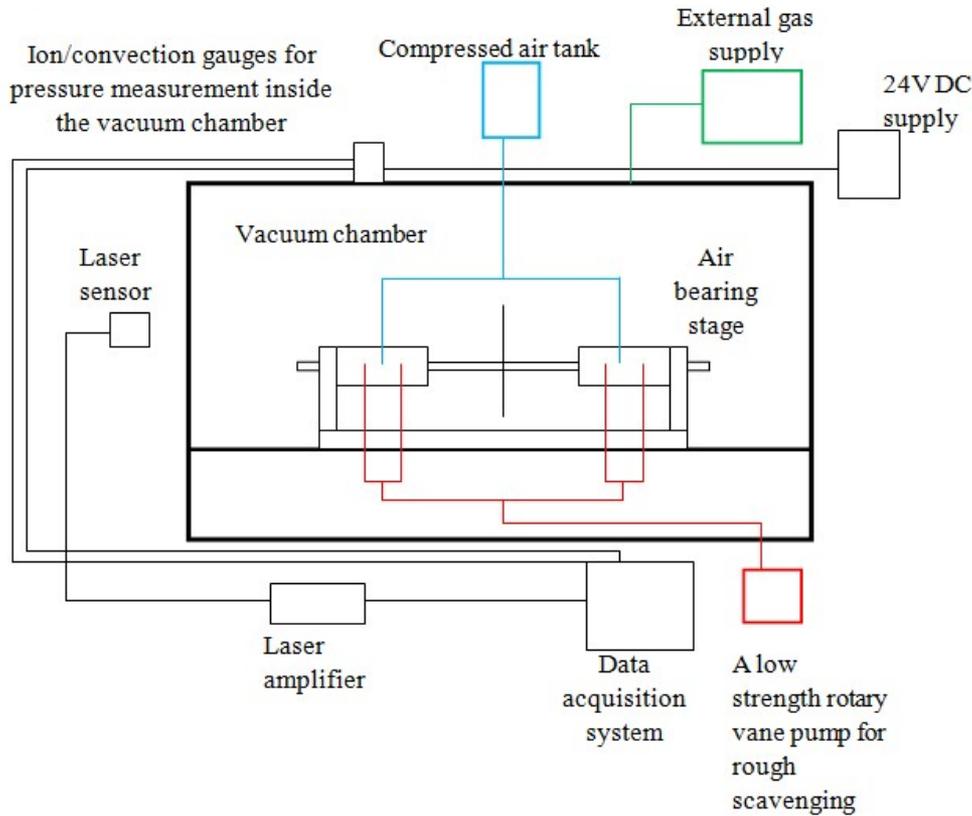
$$T = \sigma_t P_i \sqrt{\frac{m\pi}{2k_b T_i}} (R_o^4 - R_i^4) \omega = \sigma_t C_1 P_i \omega$$

- If we remove other torques:

$$\frac{d\omega}{dt} = -\frac{T}{I} = -\frac{\sigma_t P_i}{I} \sqrt{\frac{m\pi}{2k_b T_i}} (R_o^4 - R_i^4) \omega$$



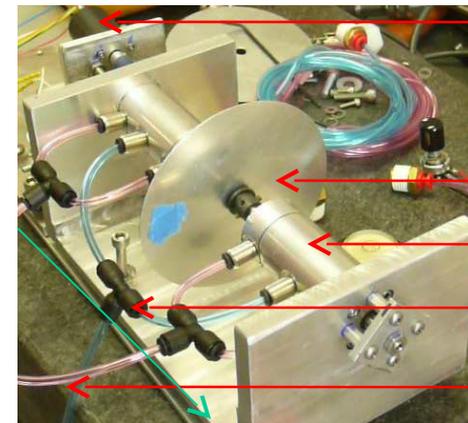
Measurement of viscous drag on a disc spinning in a low pressure gas



Schematic



Vacuum chamber



Air bearing fixture holding the test disk

- DC motor
- Aluminum test disk
- Air bushing
- Compressed air supply line
- Scavenge line to dedicated vacuum pump

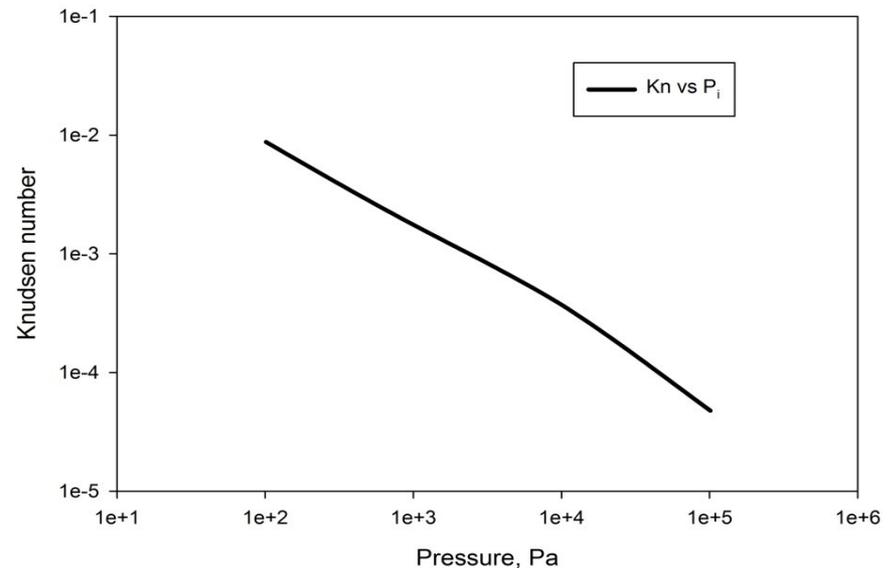
Computational Work

- Use Navier-Stokes solutions from commercial 3-D solver to provide reference solution, look for continuum breakdown. The length scale L for this flow is a gradient length scale:

$$\frac{\partial V_{mag}}{\partial z} = \frac{1}{\sqrt{V_x^2 + V_y^2 + V_z^2}} \left(V_x \frac{\partial V_x}{\partial z} + V_y \frac{\partial V_y}{\partial z} + V_z \frac{\partial V_z}{\partial z} \right)$$

$$L = \frac{V_{mag}^2}{\left(V_x \frac{\partial V_x}{\partial z} + V_y \frac{\partial V_y}{\partial z} + V_z \frac{\partial V_z}{\partial z} \right)}$$

- Continuum slip flow limit at less than 100 Pa.
- Free molecular flow limit anticipated at less than 1 Pa

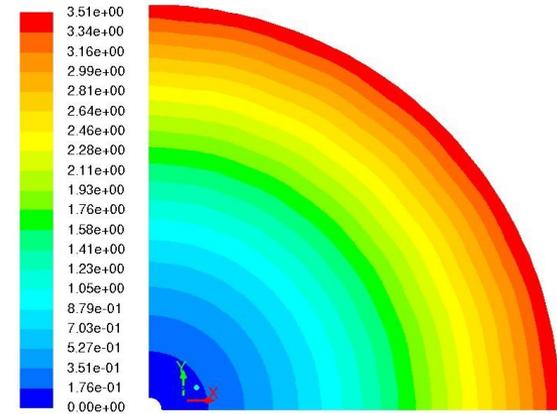


Kn variation versus chamber ambient pressure

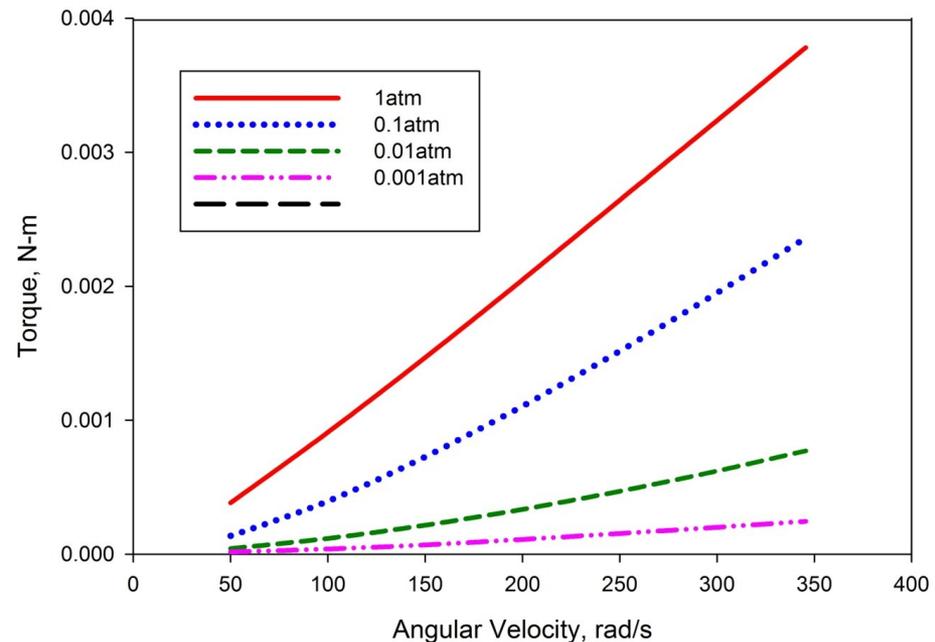
Computational Work



- Uniform wall shear stress contours on the disc surface rotating inside a cylindrical chamber of diameter 28 inches and length 28 inches
- No significant changes in wall shear stress with larger dimensions
- Additional CFD shows wall effects from chamber not a factor.



Wall shear stress, Pa



Choosing a scale for non-dimensionalizing torque



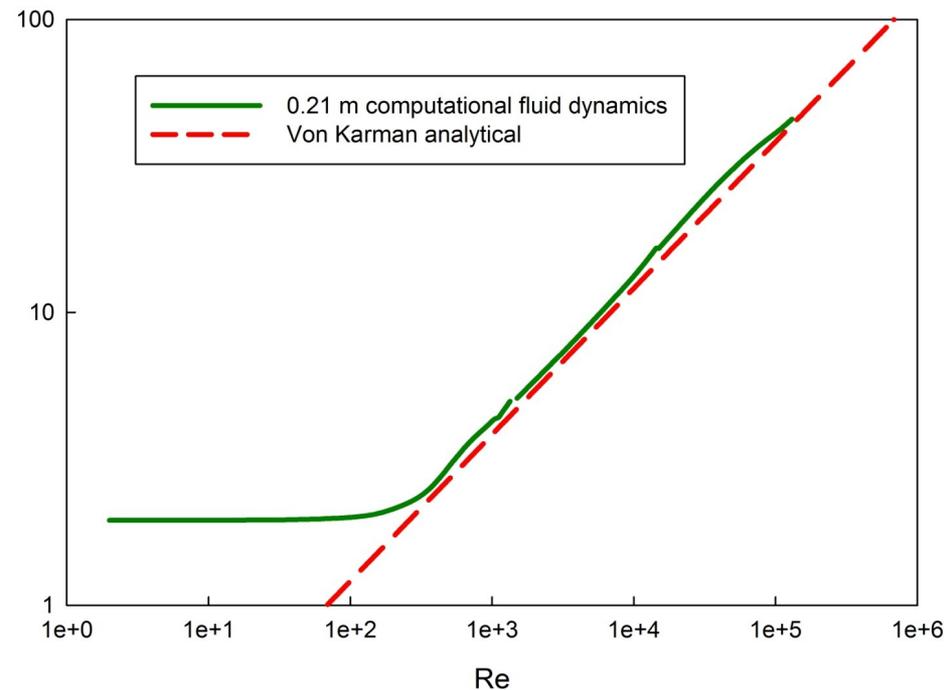
- A scale with dynamic viscosity should work well in the viscous flow regime and the non-dimensional curves should be self-similar. However, as viscosity law breaks down due to rarefied effects, self-similarity will disappear.

$$T^* = T / (\mu \omega D^3)$$

$$\text{Re} = \rho \omega D^2 / \mu$$

- At high Reynolds numbers, results agree with von Karman pump solution:

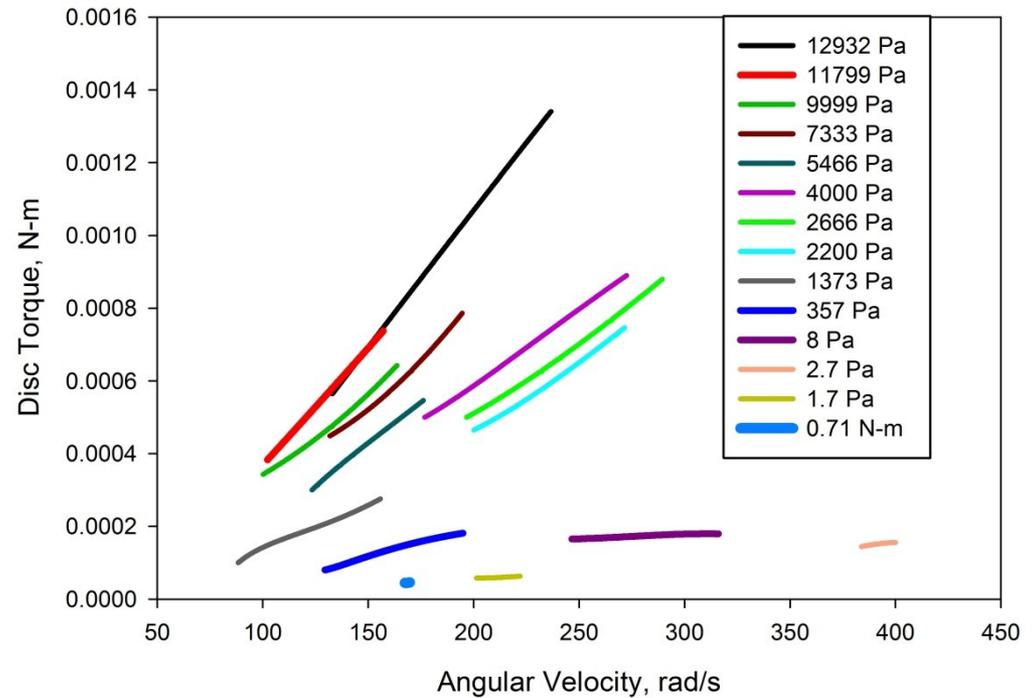
$$T^*_{\text{von-Karman}} = \frac{0.6159 (r_o^4 - r_i^4) \pi}{r_o^4} \sqrt{\text{Re}}$$



Drag Results



- Initial results from a 10 cm disk show torque proportional to rotational velocity
- Facility upgraded with larger disk, scavenging of air bearings.
- Able to get down to 1 Pa with scavenging of air bearings.
- Friction of air bearings limits accuracy of measurements.

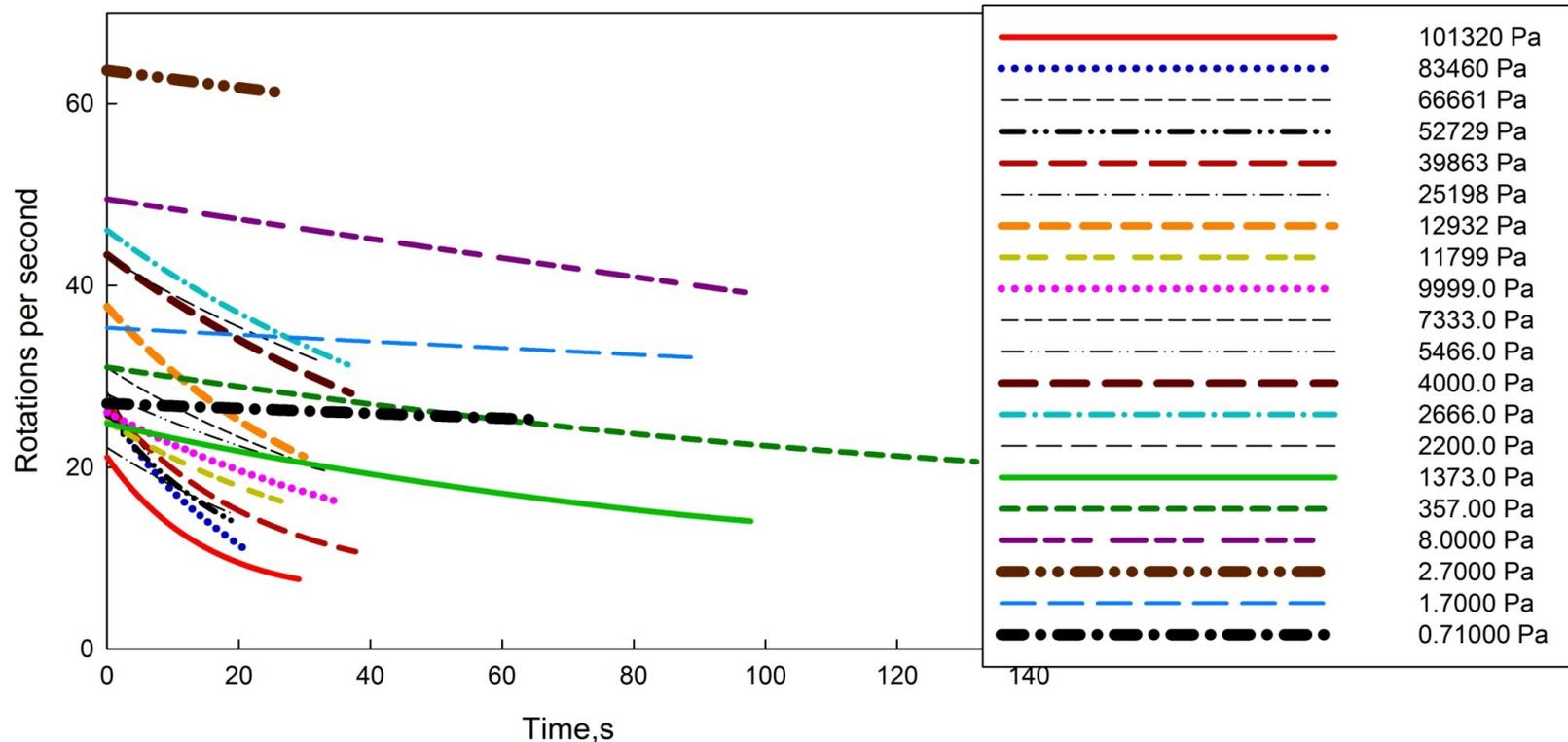


Torque versus Angular velocity.

Raw experimental data with 8.25 inch disc

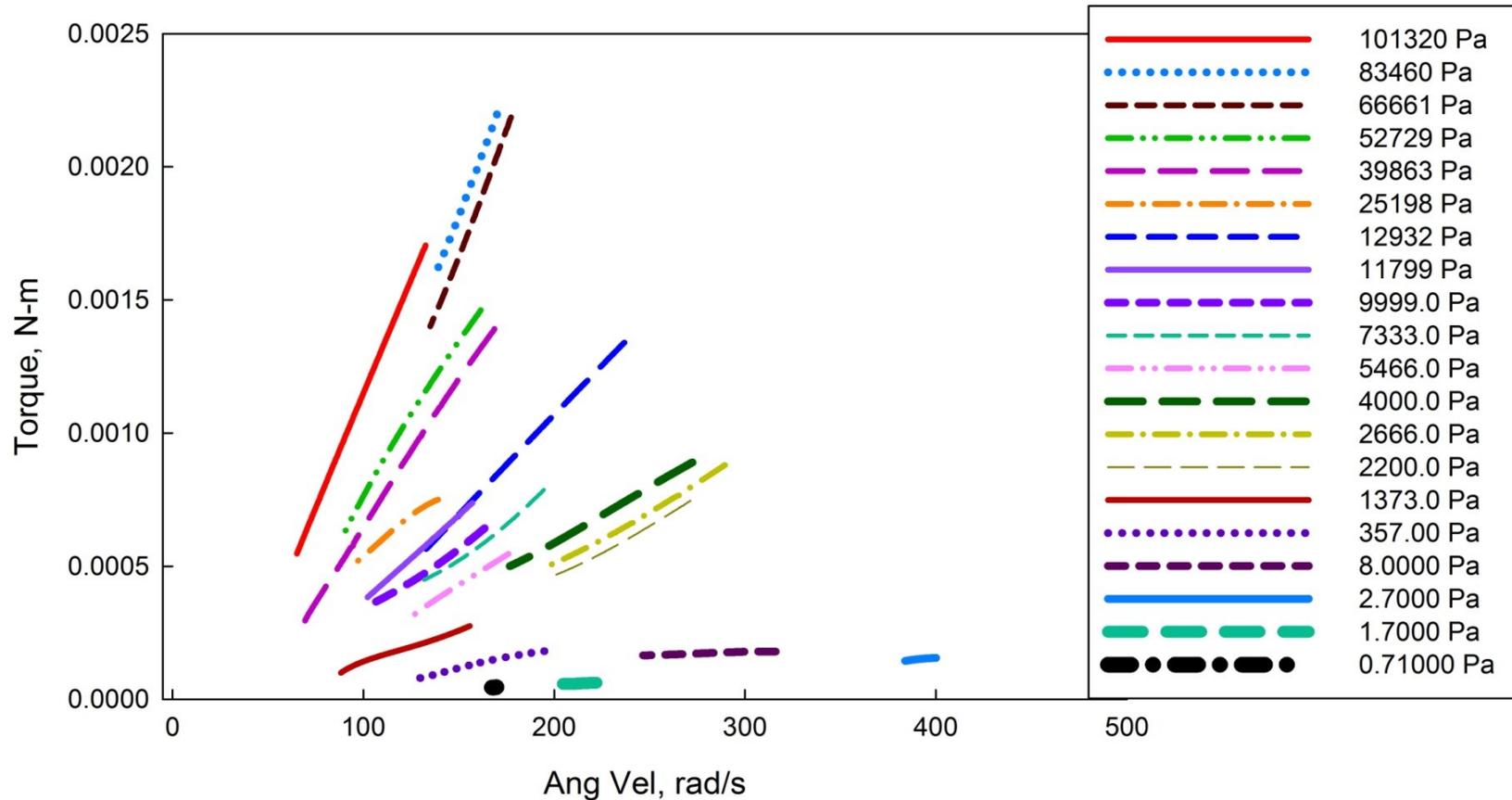


- System upgraded with scavenging to get to lower pressures (1 Pa).
- Simultaneous change to a larger (20 cm disk).



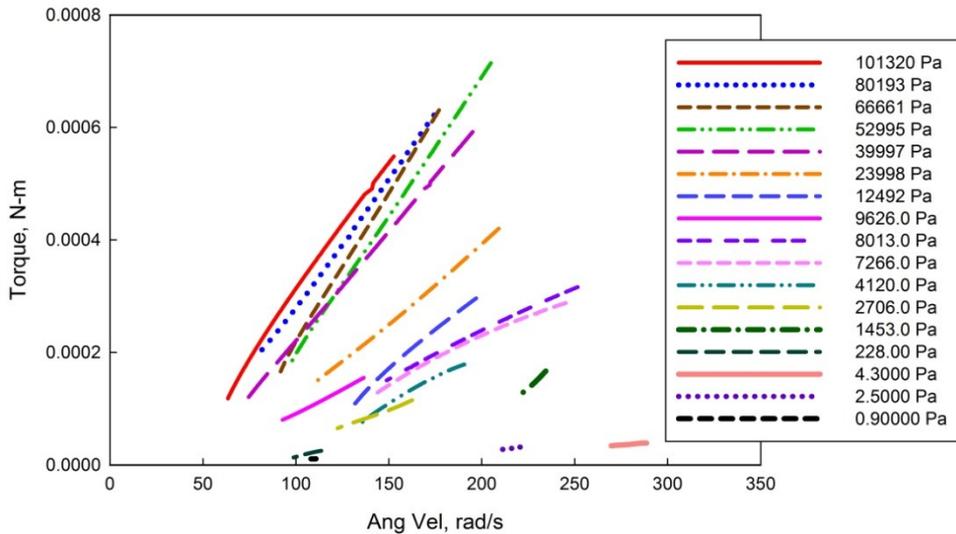
Disc rotations per second versus time

Final Results – Air versus aluminum

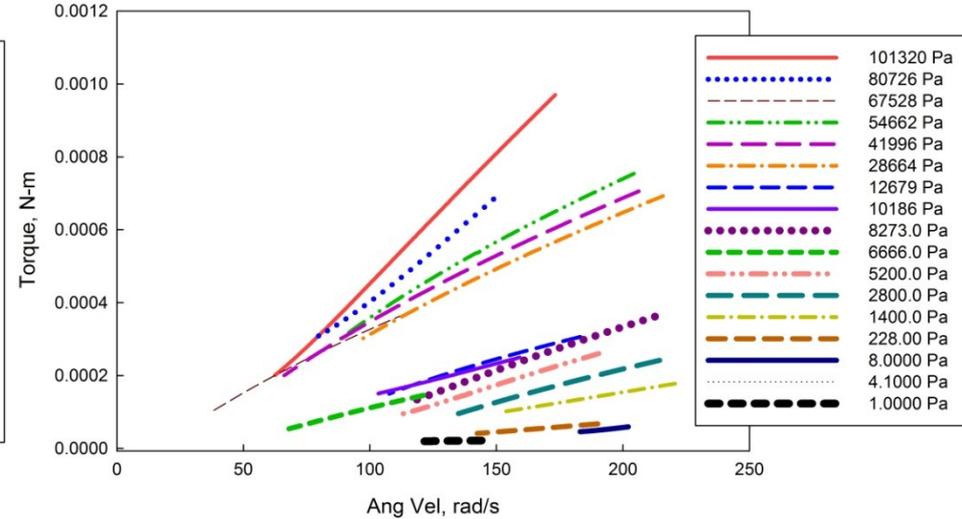


- Torque can be obtained from the spin-down time.
- Results can be corrected to remove internal friction.

Results – Torque vs Angular Velocity



6 inch disc

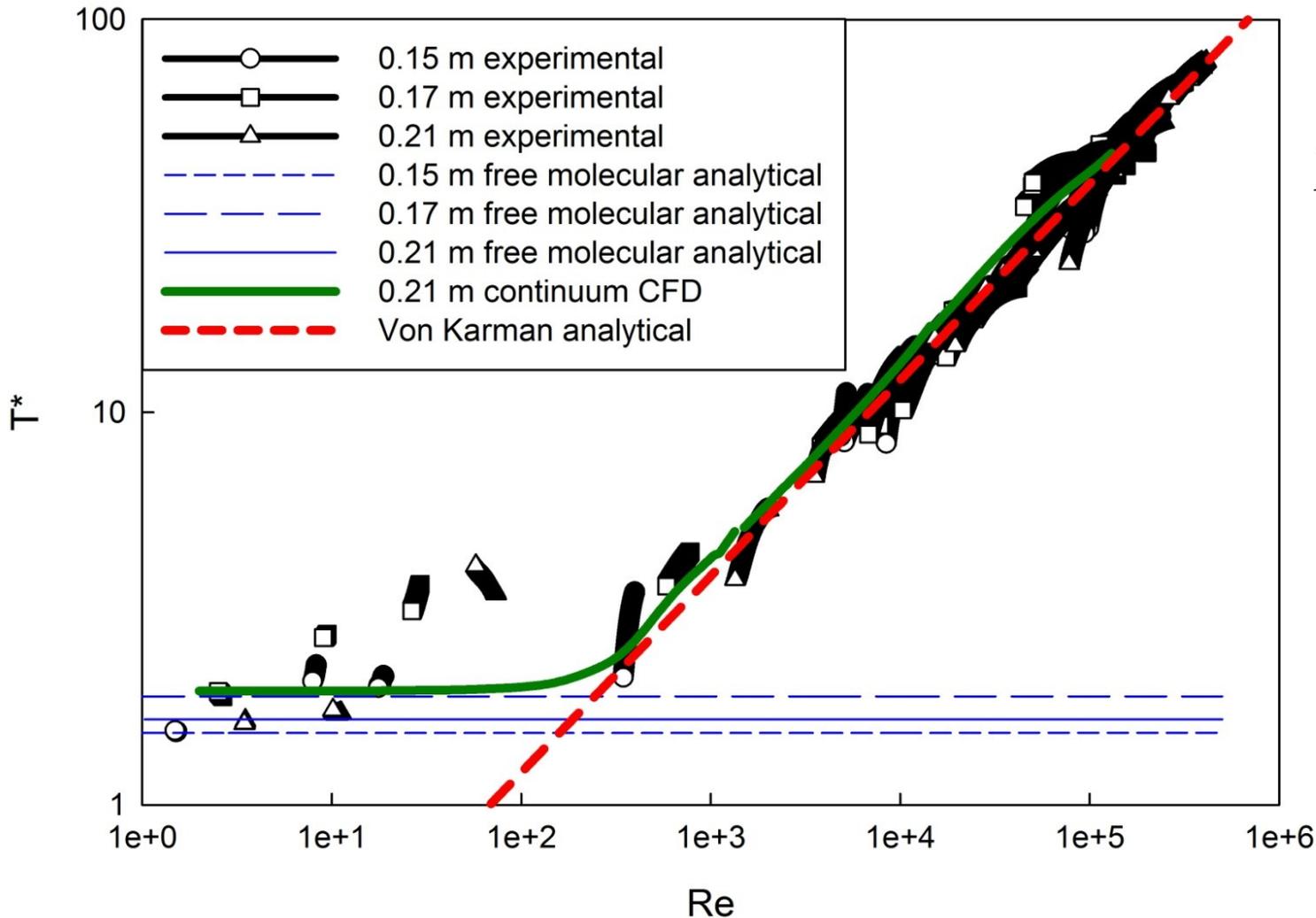


6.5 inch disc

• Experiments performed with additional sizes of aluminum discs. Torque versus angular velocity measured for a 6 inch diameter disc and a 6.5 inch diameter disc.

• Thicker disc also used to conform results independent of disc mass.

Results – Continuum Breakdown



$$T^* = \frac{T}{\mu \omega D^3}$$

$$Re = \frac{\rho \omega D^2}{\mu}$$

TMAC = 0.74 ± 2.7%

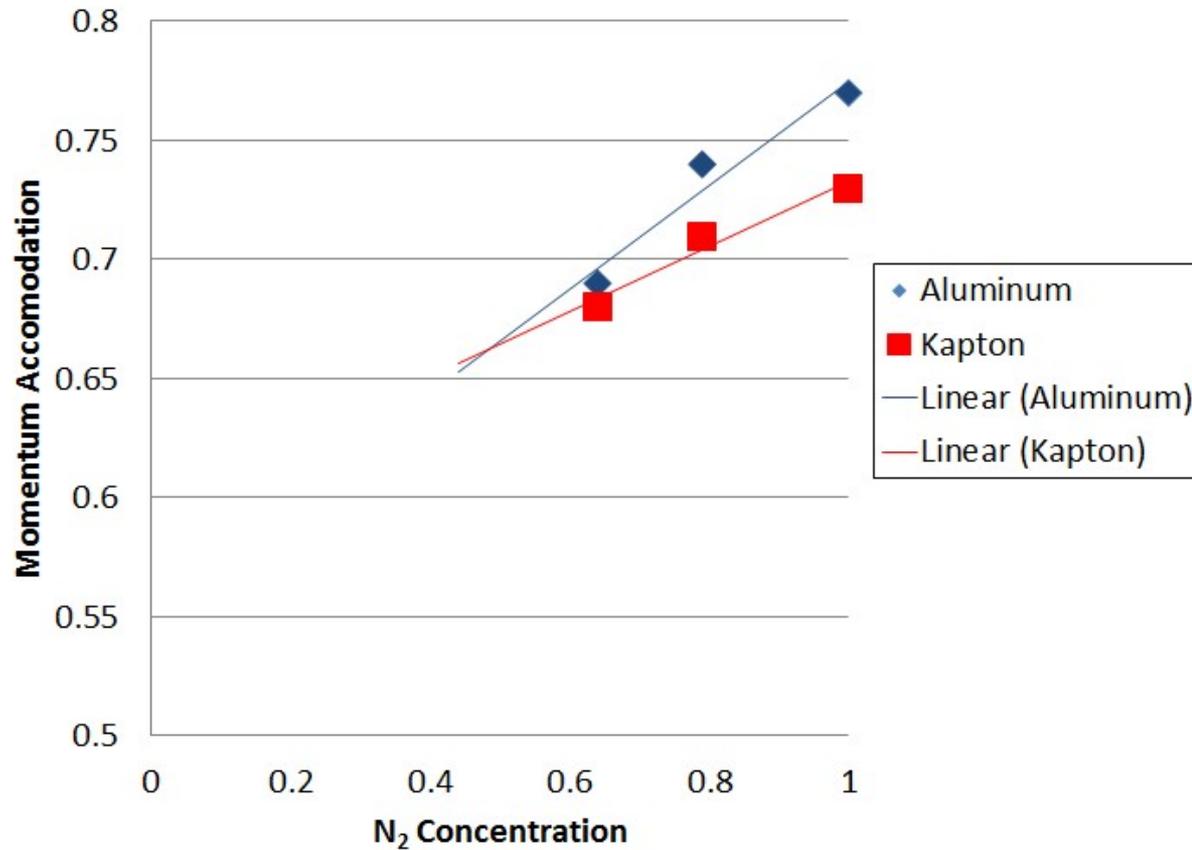
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- Measurements obtained for selected materials and gases:

Material	N₂	Air	36% O₂	Argon	CO₂
Aluminum	0.77	0.74	0.68	0.76	0.42
Titanium	0.91	0.77	-	-	-
Carbon Fiber	-	0.90	0.71	-	-
Kapton	0.73	0.71	0.68	0.60	0.57
Material	N₂	Air	36% O₂	Argon	CO₂

- Varying ratio of N₂ and O₂ suggests that there may be a mixing law, but want to avoid high O₂ mixtures for safety reasons.

Results



- Results suggest we may be able to predict effect of mixtures from individual gas values, but unable to stretch limits due to concerns about O₂.

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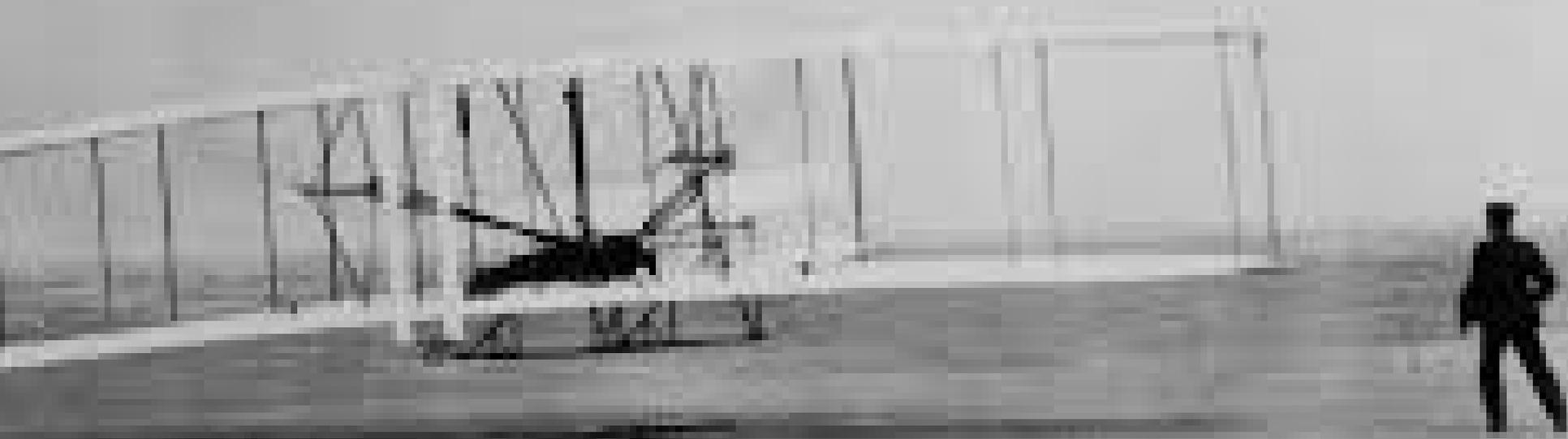
Conclusion # 1:

~~THE TRUTH IS OUT THERE~~

Experimental
Validation

Transition flow results at low speed from resonators
and rotating disk allow validation of moment method

Conclusion # 2:



For the near future, we have the same basic set of tools (measurement of force) for diagnosing flows at the micro-scale that the Wright brothers had for macro-scale aerodynamics.

Conclusion # 3:



We are not sure what happens experimentally when a gas molecule hits a surface. Until this is resolved, developing meaningful boundary conditions for moment methods will be extremely challenging⁵⁹.

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