Using real option analysis to quantify the effects of ethanol policy on production

A PDE-based stochastic control approach

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Aug 16, 2013

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1. Ethanol background

2. Assembling the model
   - The facility
   - Stochastic price model
   - Stochastic optimal control

3. Numerical results
   - Increased correlation
   - Subsidy policy

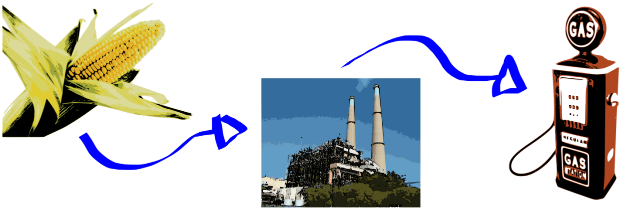
4. Policy Uncertainty
   - Policy History
   - A simple model
   - Conclusions

Ethanol biofuel production in North America has grown in popularity due to environmental and economic reasons. It has become politically attractive to subsidize biofuel programs (ethanol in particular), however subject to much policy uncertainty.

Idea

Can real options and mathematical finance aid as a tool for policy analysis?

Corn ethanol in a few words: You make money on the spread.



Assembling the model

A model for studying policy impact on ethanol production will require:

1. A model for the plant including flexibility control, capitalized construction costs, profit as a function of ethanol/corn, costs to pause/resume production
2. A stochastic model for corn and ethanol prices
3. An optimal operating rule which maximizes expected future profits

Management flexibility (control)

Depending on price conditions, management has the option to:

1. Enter into the investment with a capitalized cost of B dollars per gallon of production capacity
2. Start or stop production depending on expected profitability (i.e. to avoid running at a loss over sustained periods and to recapture profits if price become favourable again)
   - To start production on from off costs a penalty D_{01} dollars per gallon
   - To stop production incurs a penalty of D_{10}

The two main states are on (1) and off (0).

The process

When in regime i our running profit is $f_i(\dots)$

corn $\rightarrow$ ethanol + by-products

$$\underbrace{\text{profit}}_{f_1} = \underbrace{\text{yield}}_{\kappa} \ (\underbrace{\text{ethanol}}_{X_t} + \underbrace{\text{subsidy} - \text{running costs}}_{s-p=-K_1}) - \underbrace{\text{corn}}_{Y_t}$$

$$f_1 = \kappa(X_t - K_1) - Y_t$$

1. 1 bushel of corn produces κ gallons of ethanol
2. K_i is the fixed running cost per gallon in state i

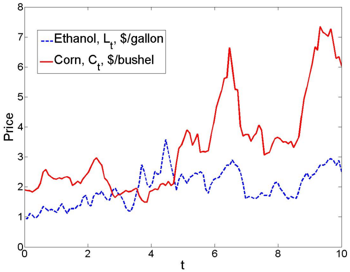
While idle, the running costs are fixed $f_0 = -\kappa K_0$.

Model parameters

Statistical tests found weak evidence for mean reversion or seasonality (*Kirby & Davison, 2010*), hence a joint GBM diffusion was chosen.

Ethanol: X_t
Corn: Y_t

$$\begin{aligned} dX_t/X_t &= \mu dt + \sigma dW_{1t} \\ dY_t/Y_t &= a dt + b dW_{2t} \\ \mathrm{Corr}[W_{1t}, W_{2t}] &= \rho \end{aligned}$$



Parameters were estimated using basic regression analysis.

Expected earnings

Total expected discounted earnings

$$J_i(t,\alpha,x,y) = E\left[\int_t^T e^{-r(s-t)} f_{I_s}(s,X_s,Y_s)ds - \sum_{k=1}^{n} e^{-r(\tau_k-t)} D_{i_{k-1},i_k} \middle| \mathcal{F}_t\right]$$

where $\alpha = (\tau_k, i_k)$ is a switching control sequence between the on and off states

- $t \leq \tau_k \leq T$ denotes an increasing sequence of stopping times (when to switch),
- and corresponding states i_k (where to switch),
- with $I_t = \sum_{0\leq k\leq n} i_k 1_{[\tau_k,\tau_{k+1})}$

We assume the facility has no salvage value at the end of its useful life $V_i(T,X_T,Y_T) = 0$

Optimal choice

The value function in state i at time t assuming subsequent optimal control is $V_i(t,x,y) = \sup_\alpha J_i(t,\alpha,x,y)$.
Dynamic programming principle:

$$\begin{aligned} V_i(t,x,y) &= \sup_\tau E\left[\int_t^\tau e^{-r(s-t)} f_i(s,X_s,Y_s)ds \right. \\ &+ \left. \max_{0\le j\le m} e^{-r(\tau-t)}\left\{V_j(\tau,X_\tau,Y_\tau) - D_{ij}\right\} 1_{\{\tau<T\}} \middle| \mathcal{F}_t \right] \end{aligned}$$

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So either (A) it is optimal to stay in the current state i and the PDE is satisfied C_i:

$$\frac{\partial V_i}{\partial t} + \mathcal{L}[V_i] + f_i(t,x,y) - rV_i = 0, \quad V_i \ge V_j - D_{ij}$$

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or (B) it is optimal to switch S_{ij}

$$V_i = \max_{0\le j\le m} V_j - D_{ij}$$

where $\mathcal{L}$ is the generator associated with the diffusion.

This is a system of free boundary PDEs (or linear complimentary problem)

Variational inequalities

$$\max\left(\underbrace{\frac{\partial V_i}{\partial t}+\mathcal{L}[V_i]+f_i(t,x,y)-rV_i}_{C_i},\ \underbrace{\max_{0\le j\le m}[(V_j-D_{ij})-V_i]}_{S_{ij}}\right)=0$$

where the generator $\mathcal{L}$ is

$$\mathcal{L}=\mu x\frac{\partial}{\partial x}+\frac{1}{2}\sigma^2x^2\frac{\partial^2}{\partial x^2}+\rho\sigma xby\frac{\partial^2}{\partial x\partial y}+\frac{1}{2}b^2y^2\frac{\partial^2}{\partial y^2}+ay\frac{\partial}{\partial y}$$

In other words:

$$PDE\le 0,\quad Constraint\le 0,\quad PDE\times constraint=0$$

Entry into investment

Suppose the firm has a finite lease time on the green field site to build the facility. The decision to enter is analogous to an American call struck at B on the potential future earnings (compound chooser option).

$$\max\left\{\frac{\partial V}{\partial t}+\mathcal{L}[V]-rV,\quad \max\left[V_1(x,y,t),V_0(x,y,t)\right]-B\right\}=0$$

Numerical method - Finite differences

We call the values at the grid points $V(x_i, y_j, t_k) = V_{i,j}^k$

- $t_k = t_0 + k\Delta t$
- $x_i = x_0 + i\Delta x$
- $y_j = y_0 + j\Delta y$

$$\frac{\partial V}{\partial x} \approx \frac{V_{i+1} - V_{i-1}}{2\Delta x}$$

$$\frac{\partial V}{\partial t} \approx \frac{V_i^{k+1} - V_i^k}{\Delta t}$$

$$\frac{V^{k+1} - V^k}{\Delta t} + \underbrace{LV^k}_{L=differentiation\ \ matrix} + f^k \leq 0$$

Leads to coupled linear complimentary problem

$$MV - b \leq 0, \quad V \geq g, \quad (MV - b)^T (V - g) = 0$$

where obstacle $g = V_* - D_*$, $b = -V^{k+1} - f^k$, $M = (\Delta t L - 1)$.

Effects of ρ: lessons from exchange options

If there are no switching costs, then this is a running exchange option:

$$V(x,y,t) = E_{x,y}\left[\int_t^T (\kappa X_s - Y_s)^+ \, ds \middle| \mathcal{F}_t\right]$$

which has solution

$$V(t,x,y) = \int_t^T e^{-r(s-t)}\left[\kappa x e^{\mu(s-t)}\Phi(q_+) - y e^{a(s-t)}\Phi(q_-)\right] ds$$

where

$$\begin{aligned} q_\pm &= \frac{\ln\left(\frac{\kappa x e^{\mu(s-t)}}{y e^{a(s-t)}}\right)}{\nu\sqrt{s-t}} \pm \frac{1}{2}\nu\sqrt{s-t} \\ \nu^2 &= \sigma^2 - 2\rho\sigma b + b^2 \end{aligned}$$

Outline of policy investigation

We investigate the effects of:

- increased correlation
- increased corn prices
- reductions in ethanol subsidy

on the following:

- plant value
- operating characteristics
- decision to enter into investment

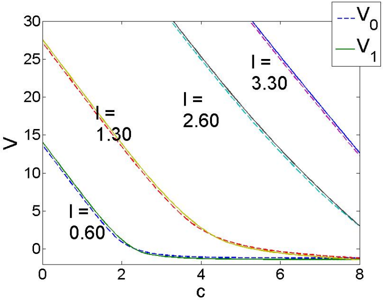


Figure: Level lines of ethanol L vs corn C.

Baseline value and switching regions

The baseline value results and effects of ρ on a 10 year horizon problem.

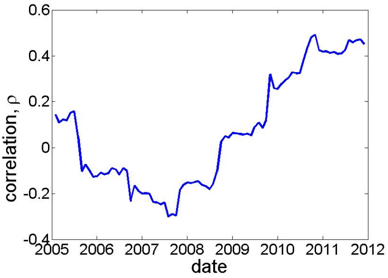


Figure: 3-year rolling correlation Jan/04–Dec/11.

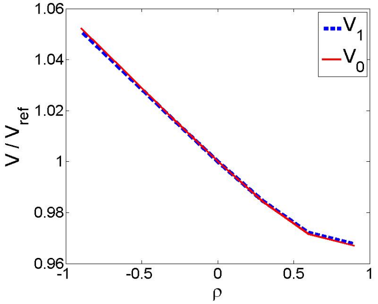


Figure: $V_i/V_{i,ref}$, the option loses much value as *rho* increases.

Loss in value and effects on operating decisions ρ

As ethanol policy induces more players into the market, the projects become less valuable due to increased correlation between corn/ethanol driven by ethanolic biofuel production demand.

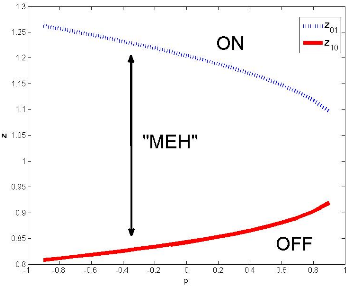


Figure: The switching boundaries vs ρ. Here we grouped $z = \frac{ethanol}{corn} = \frac{x}{y}$.

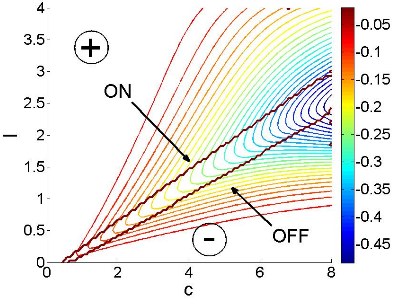


Figure: A contour plot of $\frac{\partial V_1}{\partial \rho}$.

Effects of subsidy policy on operation

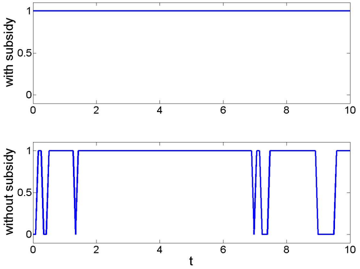


Figure: Positive profits with/without subsidy using historical price series $1_{f_1>0}$.

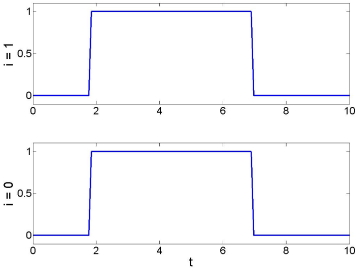


Figure: An operating signal using the historical realized price series in the absence of subsidy.

Effects of subsidy policy on entry into investment

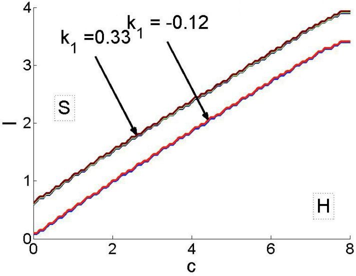
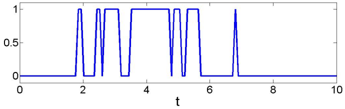


Figure: The set prices at which the investment it entered into S or defer the decision H.



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Policy uncertainty in ethanol subsidy

The level of subsidy has varied over the years. This subjects investment decisions to added uncertainty.

Act	Year	Subsidy ($/gallon)
Energy Tax Act	1978	0.40
Surface Transportation Assistance Act	1983	0.50
Tax Reform Act	1984	0.60
Omnibus Budget Reconciliation Act	1990	0.54
1998 change effective 2001	2001	0.53
1998 change again effective 2003	2003	0.52
Extension to 2007 but reduced in 2005	2005	0.51
Farm Bill	2008	0.45

Can we account for this?

It is not easily hedged, but we can try model it stochastically...

Subsidy jump process

Choose a simple Merton style jump model for the subsidy S_t

$$dS_t = (J - S_t)dq, \quad J \sim \text{logN}(a, b)$$

and dq is a Poisson process with arrival rate λ

To simplify the analysis, group $\kappa X - Y$ into Z. Choose Z to be a GBM and estimate parameters by shifting the series up by z_{min}.

$$\underbrace{\text{profit}}_{f} = \underbrace{\text{yield}}_{\kappa} \; (\underbrace{\text{ethanol}}_{X_t} + \underbrace{\text{subsidy}}_{S_t} - \underbrace{\text{running costs}}_{p}) - \underbrace{\text{corn}}_{Y_t}$$

$$f = \underbrace{\kappa X_t - Y_t}_{Z_t} - \kappa p + \kappa S_t$$

The profit function is then: $f(Z_t, S_t) = Z_t - z_{min} - \kappa(p - S_t)$

PIDVI and numerical method

The coupled control problem for the operator is given by PIDVI

$$\max\left(\frac{\partial V_i}{\partial t} + \mathcal{L}[V_i] + f_i(z,s) + \lambda E[V_i(J)] - (r+\lambda)V_i,\ \ (V_j - D_{ij}) - V_i\right) = 0$$

To account for the integral, just discretize the sum and throw it into differentiation matrix:

$$E\left[V(x,J,t)\right] = \int_0^{S_{max}} V(z,s,t)g(s)ds = \sum_{j=0}^{S_{max}} V_{i,j}^k g(j\Delta S)\Delta S$$

Value and decision boundaries

Here are the baseline cases at different subsidy levels. Recall

$$Z = \kappa X - Y$$

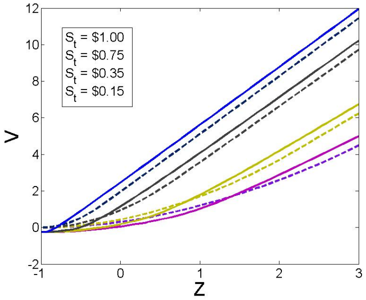

Figure: Values at different spreads and subsidies.

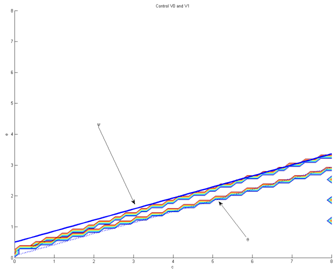

Figure: 1D switching boundaries overlaid onto 2D model: $X = \frac{1}{\kappa}(\theta + Y)$

Effects of policy uncertainty

Effects of uncertainty: Makes you switch 'sooner' and pushes value to 'expected' policy level.

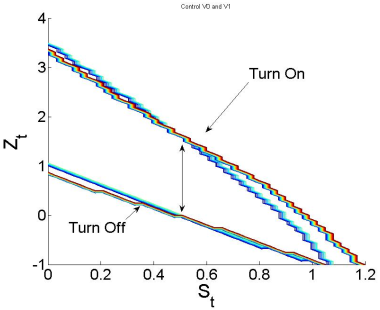

Figure: Switching decisions with and without uncertainty.

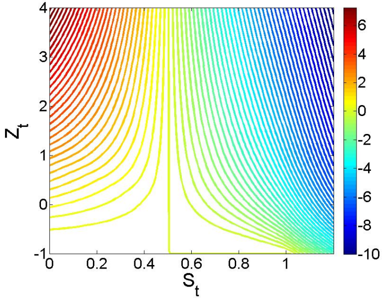

Figure: Change in value $V_{risk}(Z_t, s, t) - V_{no\ risk}(Z_t, S_t, t)$

Recap

We were able to use real options analysis and stochastic control to

- quantitatively determine how price uncertainty affects operating decisions over life of facility (i.e. optimal switching/entry times)
- measurably assess how external policy factors affect these decisions and the economic viability of the facility

Conclusion:

Real options analysis is an insightful tool for studying energy policy.

Thank you!