Focussing and normal mode scattering of the first mode internal tide by mesoscale eddy interaction

Michael Dunphy

University of Waterloo

June 12, 2013

University of Waterloo

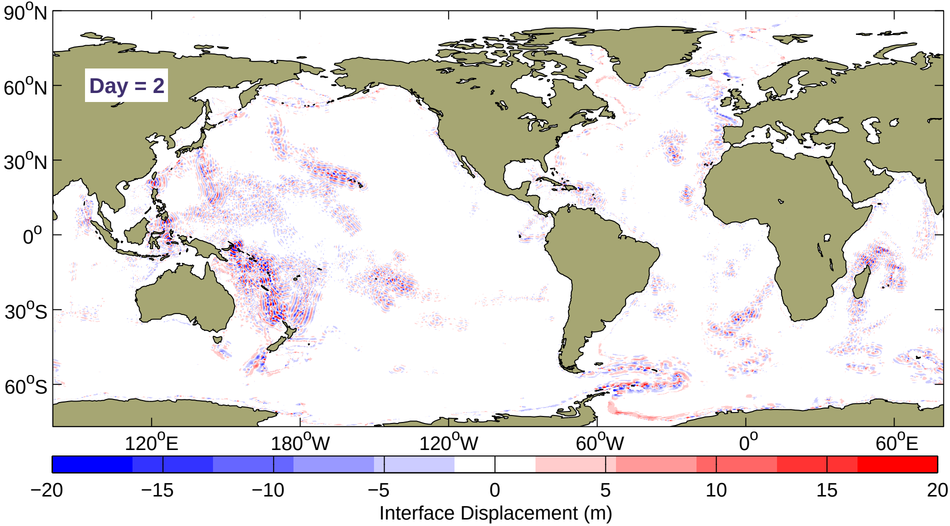
Workshop on Sub-mesoscale Ocean Processes

- There are a variety of internal wave generation mechanisms

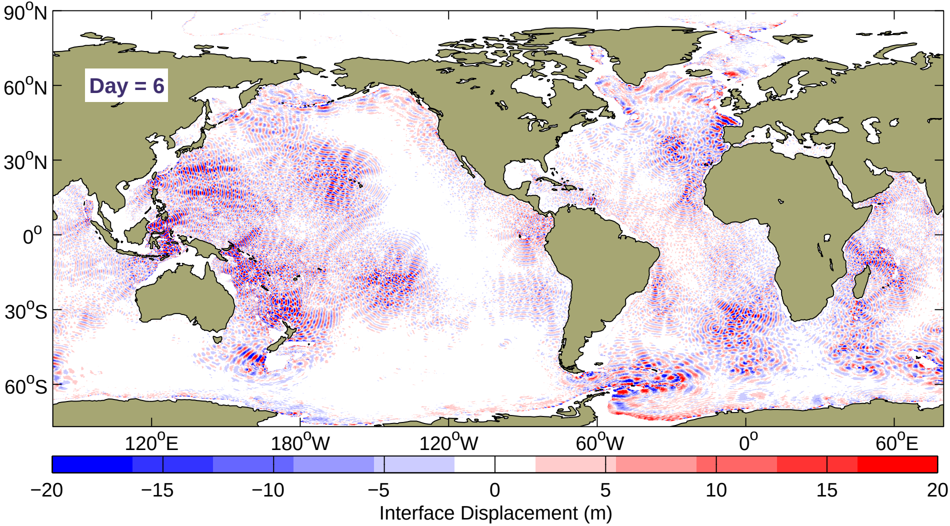
- There are a variety of internal wave generation mechanisms
- Barotropic tide-topography interaction has received a lot of attention

- There are a variety of internal wave generation mechanisms
- Barotropic tide-topography interaction has received a lot of attention
- [Bell(1975)], [Legg and Huijts(2006)], and many others
- M_2 internal tide generation is among the most prominent

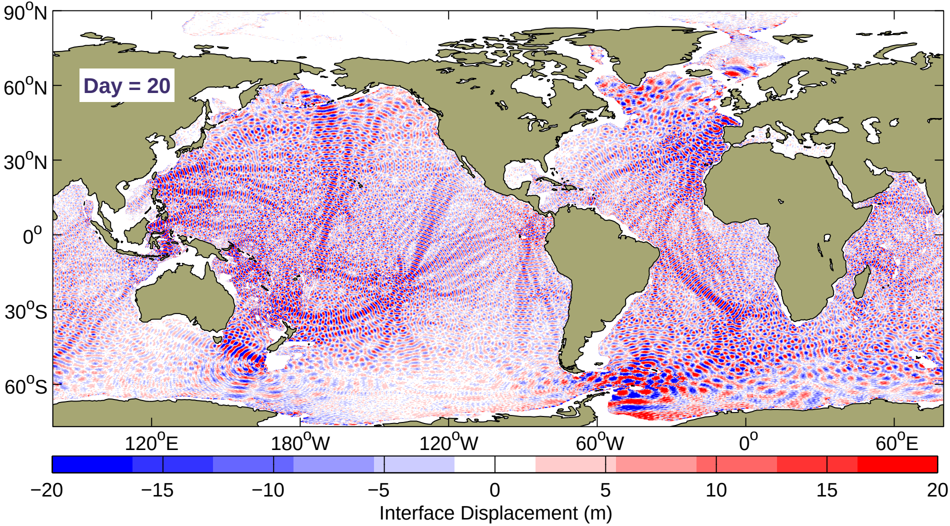
M_2 internal tide generation: (Simmons, 2004), two layer model



M_2 internal tide generation: (Simmons, 2004), two layer model



M_2 internal tide generation: (Simmons, 2004), two layer model



- The fate of the internal tide is still under investigation

- The fate of the internal tide is still under investigation
  - Local dissipation vs. propagation

- The fate of the internal tide is still under investigation
  - Local dissipation vs. propagation
  - Interaction with oceanic flows

- The fate of the internal tide is still under investigation
  - Local dissipation vs. propagation
  - Interaction with oceanic flows
  - Absorbtion into the background wavefield

- The fate of the internal tide is still under investigation
  - Local dissipation vs. propagation
  - Interaction with oceanic flows
  - Absorbtion into the background wavefield
  - Remote breaking

- The fate of the internal tide is still under investigation
    - Local dissipation vs. propagation
    - Interaction with oceanic flows
    - Absorbtion into the background wavefield
    - Remote breaking
- I'm looking at mesoscale eddy interaction

- The fate of the internal tide is still under investigation
  - Local dissipation vs. propagation
  - Interaction with oceanic flows
  - Absorbtion into the background wavefield
  - Remote breaking
- I'm looking at mesoscale eddy interaction
- This interaction is not unstudied:

- The fate of the internal tide is still under investigation
    - Local dissipation vs. propagation
    - Interaction with oceanic flows
    - Absorbtion into the background wavefield
    - Remote breaking
- I'm looking at mesoscale eddy interaction
- This interaction is not unstudied:
    - Wave-wave-vortex resonance has been looked at by [Lelong and Riley(1991)] and [Bartello(1995)]

- The fate of the internal tide is still under investigation
  - Local dissipation vs. propagation
  - Interaction with oceanic flows
  - Absorbtion into the background wavefield
  - Remote breaking
- I'm looking at mesoscale eddy interaction
- This interaction is not unstudied:
  - Wave-wave-vortex resonance has been looked at by [Lelong and Riley(1991)] and [Bartello(1995)]
  - Wave capture, [Bühler and McIntyre(2005)]

- We numerically simulate the interaction of a mode-one internal tide and an isolated mesoscale eddy

- We numerically simulate the interaction of a mode-one internal tide and an isolated mesoscale eddy
- 3D hydrostatic MITgcm with a rigid lid on an f-plane

- We numerically simulate the interaction of a mode-one internal tide and an isolated mesoscale eddy
- 3D hydrostatic MITgcm with a rigid lid on an f-plane
- The domain is a constant depth rectangular region

- We numerically simulate the interaction of a mode-one internal tide and an isolated mesoscale eddy
- 3D hydrostatic MITgcm with a rigid lid on an f-plane
- The domain is a constant depth rectangular region
- Two latitude regimes:
  - Low ($f = 0.5 \times 10^{-4}$ s^{-1}), $\approx 20^\circ$N
  - Mid ($f = 1.0 \times 10^{-4}$ s^{-1}), $\approx 43^\circ$N

- We numerically simulate the interaction of a mode-one internal tide and an isolated mesoscale eddy
- 3D hydrostatic MITgcm with a rigid lid on an f-plane
- The domain is a constant depth rectangular region
- Two latitude regimes:
  - Low ($f = 0.5 \times 10^{-4}$ s^{-1}), $\approx 20^\circ$N
  - Mid ($f = 1.0 \times 10^{-4}$ s^{-1}), $\approx 43^\circ$N

Parameter	Value
N_0	1.0×10^{-3} s^{-1}
T	44712 s (one tidal period)
ω	1.4053×10^{-4} s^{-1} (M_2)
U_t	5 cm s^{-1}
g	9.81 m s^{-2}
ρ_0	1028 kg m^{-3}
Δt	69 s

- We prescribe an isolated eddy via

$$\Psi = \psi(r)\Phi(z) = -\frac{5^{\frac{5}{2}}}{64} U_\theta L_E \mathrm{sech}^4\left(\frac{r}{L_E}\right)\Phi(z),$$

- We prescribe an isolated eddy via

$$\Psi = \psi(r)\Phi(z) = -\frac{5^{\frac{5}{2}}}{64} U_\theta L_E \operatorname{sech}^4\left(\frac{r}{L_E}\right)\Phi(z),$$

- L_E - eddy length scale
- U_θ - eddy peak velocity (at $r \approx 0.48 L_E$)

- We prescribe an isolated eddy via

$$\Psi = \psi(r)\Phi(z) = -\frac{5^{\frac{5}{2}}}{64} U_\theta L_E \operatorname{sech}^4\left(\frac{r}{L_E}\right)\Phi(z),$$

- L_E - eddy length scale
- U_θ - eddy peak velocity (at $r \approx 0.48 L_E$)
- Barotropic eddy: $\Phi(z) = 1$

- We prescribe an isolated eddy via

$$\Psi = \psi(r)\Phi(z) = -\frac{5^{\frac{5}{2}}}{64} U_\theta L_E \mathrm{sech}^4\left(\frac{r}{L_E}\right)\Phi(z),$$

- L_E - eddy length scale
- U_θ - eddy peak velocity (at $r \approx 0.48 L_E$)
- Barotropic eddy: $\Phi(z) = 1$
- Baroclinic eddy: $\Phi(z) = \cos(\pi z/H)$ - mode one

- We prescribe an isolated eddy via

$$\Psi = \psi(r)\Phi(z) = -\frac{5^{\frac{5}{2}}}{64} U_\theta L_E \text{sech}^4\left(\frac{r}{L_E}\right)\Phi(z),$$

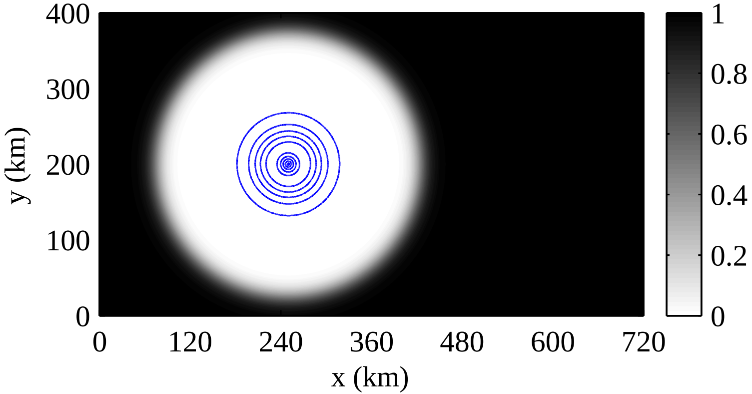
- L_E - eddy length scale
- U_θ - eddy peak velocity (at $r \approx 0.48L_E$)
- Barotropic eddy: $\Phi(z) = 1$
- Baroclinic eddy: $\Phi(z) = \cos(\pi z/H)$ - mode one
- Initialise with $(u, v) = (-\Psi_y, \Psi_x)$

- We prescribe an isolated eddy via

$$\Psi = \psi(r)\Phi(z) = -\frac{5^{\frac{5}{2}}}{64} U_\theta L_E \mathrm{sech}^4\left(\frac{r}{L_E}\right)\Phi(z),$$

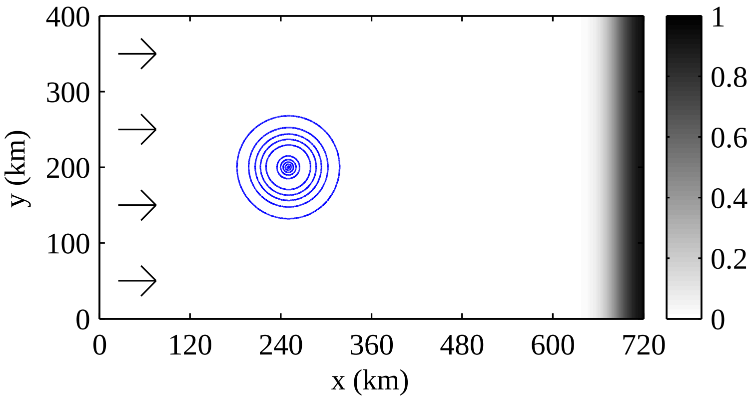
- L_E - eddy length scale
- U_θ - eddy peak velocity (at $r \approx 0.48 L_E$)
- Barotropic eddy: $\Phi(z) = 1$
- Baroclinic eddy: $\Phi(z) = \cos(\pi z/H)$ - mode one
- Initialise with $(u, v) = (-\Psi_y, \Psi_x)$
- Use cyclo-geostrophic and hydrostatic balances to find ρ'

- We let the eddy adjust for 5 tidal periods, $0 \leq t \leq 5T$
- Shaded region is relaxed to no-flow



- Then we force a mode-one internal tide at the west boundary

$$u(x=0,z,t) = U_t \sin(\omega(t-5T))\cos\left(\frac{\pi z}{H}\right) R(t-5T),$$
$$5T \le t \le 30T,$$



Parameter	Value
Barotropic cases:	
L × W × H	600 km × 800 km × 5000 m
$N_x \times N_y \times N_z$	1200 × 1600 × 25
$\Delta x \times \Delta y \times \Delta z$	0.5 km × 0.5 km × 200 m
eddy centre	$(x_c, y_c) = (250, 400)$ km

L_E \ U_θ	30 cm/s	45 cm/s	60 cm/s	75 cm/s	90 cm/s
20 km		X			
30 km	X	X	X	X	X
40 km		X			
50 km		X			

Parameter	Value
Baroclinic cases:	
L $\times$ W $\times$ H	720 km $\times$ 400 km $\times$ 5000 m
$N_x \times N_y \times N_z$	1440 $\times$ 800 $\times$ 50
$\Delta x \times \Delta y \times \Delta z$	0.5 km $\times$ 0.5 km $\times$ 100 m
eddy centre	$(x_c, y_c) = (250, 200)$ km

U_θ / L_E	30 cm/s	45 cm/s	60 cm/s
15 km	X		
20 km	X	X	
25 km	X	X	X
30 km	X	X	X
35 km	X	X	X
40 km	X	X	X
45 km	X	X	X
50 km	X	X	X
55 km	X	X	X

- Constant N yields vertical mode solutions following cosine/sine

$$\{\vec{u}_h, p'\} = \{\vec{u}_{h_0}, p'_0\}(x, y, t) + \sum_{n=1}^{\infty} \{\vec{u}_{h_n}, p'_n\}(x, y, t)\cos(m_n z),$$

$$\{w, \rho'\} = \sum_{n=1}^{\infty} \{w_n, \rho'_n\}(x, y, t)\sin(m_n z),$$

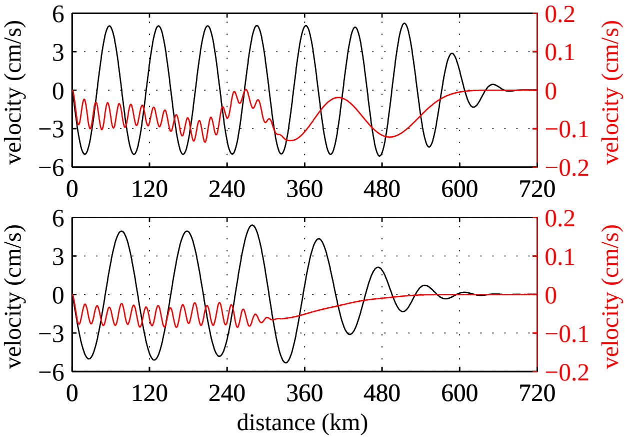
where $m_n = \frac{n\pi}{H}$.

- We compute the coefficients from the flow fields by

$$\{\vec{u}_{h_0}, p'_0\} = \frac{1}{H}\int_{-H}^{0} \{\vec{u}_h, p'\}\,\mathrm{d}z,$$

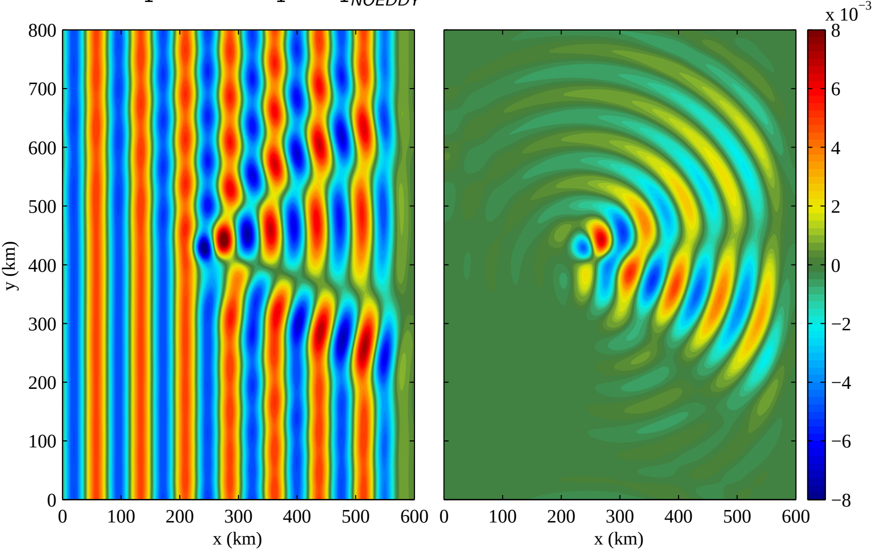
$$\{\vec{u}_{h_n}, p'_n\} = \frac{2}{H}\int_{-H}^{0} \{\vec{u}_h, p'\}\cos(m_n z)\,\mathrm{d}z,$$

$$\{w_n, \rho'_n\} = \frac{2}{H}\int_{-H}^{0} \{w, \rho'\}\sin(m_n z)\,\mathrm{d}z.$$

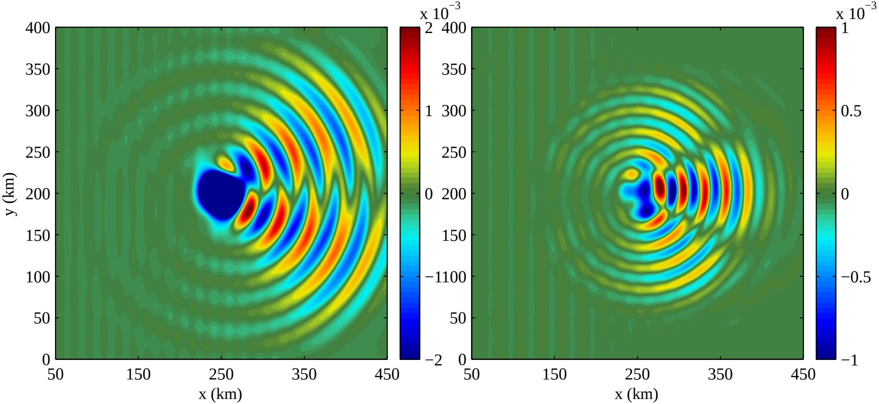


- Top: low latitude - PSI
- Bottom: mid latitude - no PSI

- Low latitude, $L_E = 50$ km, $U_\theta = 45$ cm/s, $t = 16T$
- Left: ρ_1', Right: $\rho_1' - \rho'_{1_{NOEDDY}}$



- Density perturbation at modes 2 and 3
- Low f, $L_E = 35$ km, $U_\theta = 45$ cm/s, $t = 16T$



- Review of the total pseudoenergy budget derivation

- Review of the total pseudoenergy budget derivation
- Start with the hydrostatic Boussinesq equations

$$\frac{\partial \vec{u}_h}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u}_h + f \times \vec{u}_h = \frac{-1}{\rho_0}\vec{\nabla}_h p',$$

$$\epsilon_{nh}\left(\frac{\partial w}{\partial t} + (\vec{u}\cdot\vec{\nabla})w\right) = -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} - \frac{g\rho'}{\rho_0},$$

$$\frac{\partial \rho'}{\partial t} + (\vec{u}\cdot\vec{\nabla})\rho' = \frac{\rho_0 N_0^2}{g}w,$$

where $p = \bar{p}(z) + p'$ and $\rho = \rho_0 + \bar{\rho_1}(z) + \rho'$.

- Review of the total pseudoenergy budget derivation
- Start with the hydrostatic Boussinesq equations

$$\frac{\partial \vec{u}_h}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u}_h + f \times \vec{u}_h = \frac{-1}{\rho_0}\vec{\nabla}_h p',$$

$$\epsilon_{nh}\left(\frac{\partial w}{\partial t} + (\vec{u} \cdot \vec{\nabla})w\right) = -\frac{1}{\rho_0}\frac{\partial p'}{\partial z} - \frac{g\rho'}{\rho_0},$$

$$\frac{\partial \rho'}{\partial t} + (\vec{u} \cdot \vec{\nabla})\rho' = \frac{\rho_0 N_0^2}{g} w,$$

where $p = \bar{p}(z) + p'$ and $\rho = \rho_0 + \bar{\rho_1}(z) + \rho'$.

- Hydrostatic approximation sets $\epsilon_{nh} = 0$

- Dot product between $\rho_0 \vec{u}$ and the momentum equations

- Dot product between $\rho_0 \vec{u}$ and the momentum equations
- Multiply density equation by $\dfrac{g^2 \rho'}{\rho_0 N_0^2}$

- Dot product between $\rho_0 \vec{u}$ and the momentum equations
- Multiply density equation by $\dfrac{g^2 \rho'}{\rho_0 N_0^2}$
- Add the results, integrate over a volume, use some algebra, we get

$$\frac{\mathrm{d}}{\mathrm{d}t} P + W + K_f + A_f = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P + W + K_f + A_f = 0,$$

where

$$P = \iiint_V \rho_0 \frac{1}{2}(u^2 + v^2) + \frac{g^2 \rho'^2}{2N^2\rho_0}\,\mathrm{d}V \text{ (total pseudo-energy)},$$

$$W = \iint_{\delta V} p' \vec{u} \cdot \hat{n}\,\mathrm{d}S \text{ (linear energy flux)},$$

$$K_f = \iint_{\delta V} \frac{\rho_0}{2} \vec{u} \cdot \hat{n}(u^2 + v^2)\,\mathrm{d}S \text{ (nonlinear flux of kinetic energy)},$$

$$A_f = \frac{g^2}{2\rho_0 N^2} \iint_{\delta V} \rho'^2 \vec{u} \cdot \hat{n}\,\mathrm{d}S$$

(nonlinear flux of available potential energy).

- We wish to have an energy budget for each vertical mode

- We wish to have an energy budget for each vertical mode
- Recall the cosine and sine series:

$$\{\vec{u}_h, p'\} = \{\vec{u}_{h_0}, p_0'\}(x,y,t) + \sum_{n=1}^{\infty} \{\vec{u}_{h_n}, p_n'\}(x,y,t)\cos(m_n z),$$

$$\{w, \rho'\} = \sum_{n=1}^{\infty} \{w_n, \rho_n'\}(x,y,t)\sin(m_n z),$$

where $m_n = \frac{n\pi}{H}$.

- We wish to have an energy budget for each vertical mode
- Recall the cosine and sine series:

$$\{\vec{u}_h, p'\} = \{\vec{u}_{h_0}, p'_0\}(x, y, t) + \sum_{n=1}^{\infty} \{\vec{u}_{h_n}, p'_n\}(x, y, t) \cos(m_n z),$$

$$\{w, \rho'\} = \sum_{n=1}^{\infty} \{w_n, \rho'_n\}(x, y, t) \sin(m_n z),$$

where $m_n = \frac{n\pi}{H}$.

- These series are substituted into the horizontal momentum and density equations

- We wish to have an energy budget for each vertical mode
- Recall the cosine and sine series:

$$\{\vec{u}_h, p'\} = \{\vec{u}_{h_0}, p'_0\}(x, y, t) + \sum_{n=1}^{\infty} \{\vec{u}_{h_n}, p'_n\}(x, y, t)\cos(m_n z),$$

$$\{w, \rho'\} = \sum_{n=1}^{\infty} \{w_n, \rho'_n\}(x, y, t)\sin(m_n z),$$

where $m_n = \frac{n\pi}{H}$.

- These series are substituted into the horizontal momentum and density equations
- Then we collect terms at each vertical mode

- The linear terms are straightforward,

$$\frac{\partial \vec{u}_h}{\partial t} = \frac{\partial \vec{u}_{h_0}}{\partial t} + \sum_{n=1}^{\infty} \frac{\partial \vec{u}_{h_n}}{\partial t} \cos(m_n z),$$

$$\vec{f} \times \vec{u}_h = \vec{f} \times \vec{u}_{h_0} + \sum_{n=1}^{\infty} \vec{f} \times \vec{u}_{h_n} \cos(m_n z),$$

$$\frac{-1}{\rho_0} \nabla_h p' = \frac{-1}{\rho_0} \nabla_h p'_0 + \frac{-1}{\rho_0} \sum_{n=1}^{\infty} \nabla_h p'_n \cos(m_n z)$$

- The linear terms are straightforward,

$$\frac{\partial \vec{u}_h}{\partial t} = \frac{\partial \vec{u}_{h_0}}{\partial t} + \sum_{n=1}^{\infty} \frac{\partial \vec{u}_{h_n}}{\partial t} \cos(m_n z),$$

$$\vec{f} \times \vec{u}_h = \vec{f} \times \vec{u}_{h_0} + \sum_{n=1}^{\infty} \vec{f} \times \vec{u}_{h_n} \cos(m_n z),$$

$$\frac{-1}{\rho_0} \nabla_h p' = \frac{-1}{\rho_0} \nabla_h p'_0 + \frac{-1}{\rho_0} \sum_{n=1}^{\infty} \nabla_h p'_n \cos(m_n z)$$

- The nonlinear terms require a bit of work

$$(\vec{u} \cdot \vec{\nabla}) \vec{u}_h = \vec{N}^u = \vec{N}^u_0 + \sum_{n=1}^{\infty} \vec{N}^u_n \cos(m_n z),$$

- Copious use of trig substitution yields

$$\vec{N}_0^u = (\vec{u}_{h_0}\cdot\vec{\nabla}_h)\vec{u}_{h_0} + \frac{1}{2}\sum_{i=1}^{\infty}(\vec{u}_{h_i}\cdot\vec{\nabla}_h)\vec{u}_{h_i} - \frac{1}{2}\sum_{i=1}^{\infty} w_i\vec{u}_{h_i}m_i,$$

$$\vec{N}_1^u = \left[\left((\vec{u}_{h_0}\cdot\vec{\nabla}_h)\vec{u}_{h_1} + (\vec{u}_{h_1}\cdot\vec{\nabla}_h)\vec{u}_{h_0}\right) + \frac{1}{2}\sum_{i=1}^{\infty}\left((\vec{u}_{h_i}\cdot\vec{\nabla}_h)\vec{u}_{h_{i+1}} + (\vec{u}_{h_{i+1}}\cdot\vec{\nabla}_h)\vec{u}_{h_i}\right)\right]$$
$$- \frac{1}{2}\left[\sum_{i=1}^{\infty} w_i\vec{u}_{h_{i+1}}m_{i+1} + w_{i+1}\vec{u}_{h_i}m_i\right],$$

$$\vec{N}_2^u = \left[\left((\vec{u}_{h_0}\cdot\vec{\nabla}_h)\vec{u}_{h_2} + (\vec{u}_{h_2}\cdot\vec{\nabla}_h)\vec{u}_{h_0}\right) + \frac{1}{2}(\vec{u}_{h_1}\cdot\vec{\nabla}_h)\vec{u}_{h_1}\right.$$
$$\left. + \frac{1}{2}\sum_{i=1}^{\infty}\left((\vec{u}_{h_i}\cdot\vec{\nabla}_h)\vec{u}_{h_{i+2}} + (\vec{u}_{h_{i+2}}\cdot\vec{\nabla}_h)\vec{u}_{h_i}\right)\right]$$
$$+ \frac{1}{2}\left[w_1\vec{u}_{h_1}m_1 - \sum_{i=1}^{\infty}(w_i\vec{u}_{h_{i+2}}m_{i+2} + w_{i+2}\vec{u}_{h_i}m_i)\right]$$

$$\vec{N}_3^u = \ldots.$$

Finally, the projected horizontal momentum equation is written as as sum over modes

$$\vec{M}_0 + \sum_{n=1}^{\infty} \vec{M}_n \cos(m_n z) = 0,$$

Finally, the projected horizontal momentum equation is written as as sum over modes

$$\vec{M}_0 + \sum_{n=1}^{\infty} \vec{M}_n \cos(m_n z) = 0,$$

where the coefficients of each mode sum to zero,

$$\vec{M}_0 = \frac{\partial \vec{u}_{h_0}}{\partial t} + N_0^u + \vec{f} \times \vec{u}_{h_0} + \frac{1}{\rho_0} \vec{\nabla}_h p_0' = 0,$$

$$\vec{M}_n = \frac{\partial \vec{u}_{h_n}}{\partial t} + N_n^u + \vec{f} \times \vec{u}_{h_n} + \frac{1}{\rho_0} \vec{\nabla}_h p_n' = 0, \qquad n = 1, 2, 3, \ldots$$

Finally, the projected horizontal momentum equation is written as as sum over modes

$$\vec{M}_0 + \sum_{n=1}^{\infty} \vec{M}_n \cos(m_n z) = 0,$$

where the coefficients of each mode sum to zero,

$$\vec{M}_0 = \frac{\partial \vec{u}_{h_0}}{\partial t} + N_0^u + \vec{f} \times \vec{u}_{h_0} + \frac{1}{\rho_0}\vec{\nabla}_h p_0' = 0,$$

$$\vec{M}_n = \frac{\partial \vec{u}_{h_n}}{\partial t} + N_n^u + \vec{f} \times \vec{u}_{h_n} + \frac{1}{\rho_0}\vec{\nabla}_h p_n' = 0, \qquad n = 1, 2, 3, \ldots$$

A similar procedure is used for the density equation,

$$\sum_{n=1}^{\infty} D_n \sin(m_n z) = 0, \text{ which gives}$$

$$D_n = \frac{\partial \rho_n'}{\partial t} + N_n^\rho - \frac{\rho_0 N_0^2}{g} w_n = 0, \qquad n = 1, 2, 3, \ldots$$

We take the dot product,

$$(\rho_0 H)\vec{u}_{h_0} \cdot \vec{M}_0 = 0$$

We take the dot product,

$$(\rho_0 H)\vec{u}_{h_0} \cdot \vec{M}_0 = 0$$

then integrate over an area, employ some algebra, to get

$$\frac{\mathrm{d}}{\mathrm{d}t} K_0 + W_0 + S_0 = 0,$$

We take the dot product,

$$(\rho_0 H)\vec{u}_{h_0} \cdot \vec{M}_0 = 0$$

then integrate over an area, employ some algebra, to get

$$\frac{\mathrm{d}}{\mathrm{d}t} K_0 + W_0 + S_0 = 0,$$

where

$$K_0 = \rho_0 \frac{H}{2} \iint\limits_A (u_0^2 + v_0^2)\,\mathrm{d}A$$ is total barotropic kinetic energy,

$$W_0 = H \oint\limits_{\delta A} (\vec{u}_{h_0} \cdot \hat{n}) p_0' \,\mathrm{d}S$$ is the linear barotropic energy flux, and

$$S_0 = \rho_0 H \iint\limits_A \vec{u}_{h_0} \cdot \vec{N}_0^u \,\mathrm{d}A$$ is the nonlinear barotropic energy sink.

A similar procedure yields the n-th baroclinic mode pseudo-energy budget,

$$\frac{\mathrm{d}}{\mathrm{d}t} P_n + W_n + S_n = 0, \quad \text{where}$$

A similar procedure yields the *n*-th baroclinic mode pseudo-energy budget,

$$\frac{\mathrm{d}}{\mathrm{d}t} P_n + W_n + S_n = 0, \quad \text{where}$$

$$P_n = \rho_0 \frac{H}{4} \iint_A (u_n^2 + v_n^2)\,\mathrm{d}A + \frac{Hg^2}{4\rho_0 N_0^2} \iint_A \rho_n'^2\,\mathrm{d}A$$

is the total pseudo energy at mode-n,

$$W_n = \frac{H}{2} \oint_{\delta A} (\vec{u}_{h_n} \cdot \hat{n}) p_n'\,\mathrm{d}S,$$

is the linear baroclinic energy flux at mode-n, and

$$S_n = \frac{\rho_0 H}{2} \iint_A \vec{u}_{h_n} \cdot \vec{N}_n^u\,\mathrm{d}A + \frac{Hg^2}{2\rho_0 N_0^2} \iint_A \rho_n N_n^\rho\,\mathrm{d}A,$$

is the nonlinear sink of pseudo-energy at mode-n.

The budgets are

$$\text{Barotropic kinetic energy:} \quad \frac{\mathrm{d}K_0}{\mathrm{d}t} + W_0 + S_0 = 0$$

$$\text{Baroclinic pseudo-energy:} \quad \frac{\mathrm{d}P_n}{\mathrm{d}t} + W_n + S_n = 0$$

$$\text{Total pseudo-energy:} \quad \frac{\mathrm{d}P}{\mathrm{d}t} + W + K_f + A_f = 0$$

The budgets are

$$\begin{aligned}
\text{Barotropic kinetic energy:}\quad & \frac{\mathrm{d}K_0}{\mathrm{d}t} + W_0 + S_0 &&= 0 \\
\text{Baroclinic pseudo-energy:}\quad & \frac{\mathrm{d}P_n}{\mathrm{d}t} + W_n + S_n &&= 0 \\
\text{Total pseudo-energy:}\quad & \frac{\mathrm{d}P}{\mathrm{d}t} + W + K_f + A_f &&= 0
\end{aligned}$$

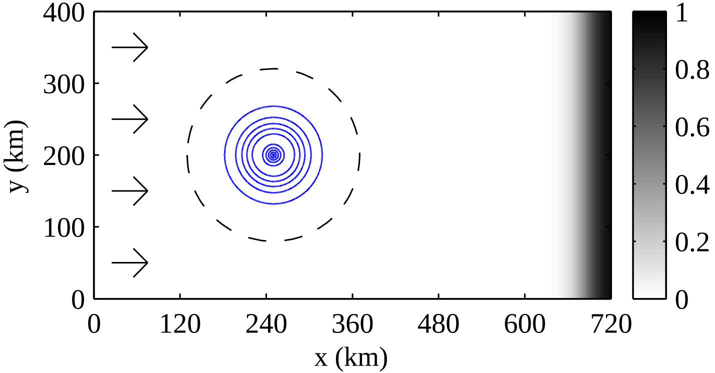
which, as you might expect, sum via

$$\frac{\mathrm{d}}{\mathrm{d}t}K_0 + \sum_{n=1}^{\infty} \frac{\mathrm{d}}{\mathrm{d}t}P_n = \frac{\mathrm{d}}{\mathrm{d}t}P,$$

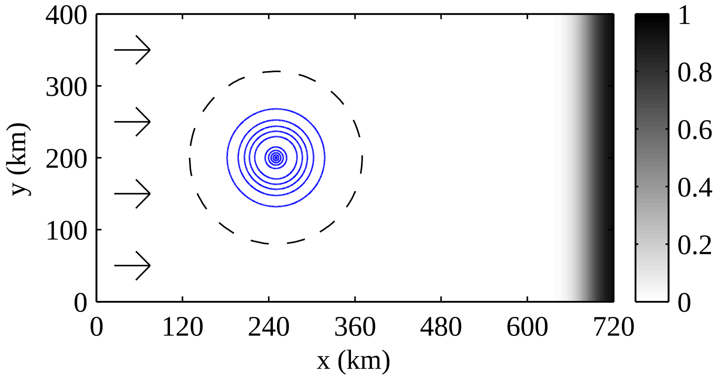
$$W_0 + \sum_{n=1}^{\infty} W_n = W,$$

$$S_0 + \sum_{n=1}^{\infty} S_n = K_f + A_f$$

- The energy budget is computed inside the dashed circle

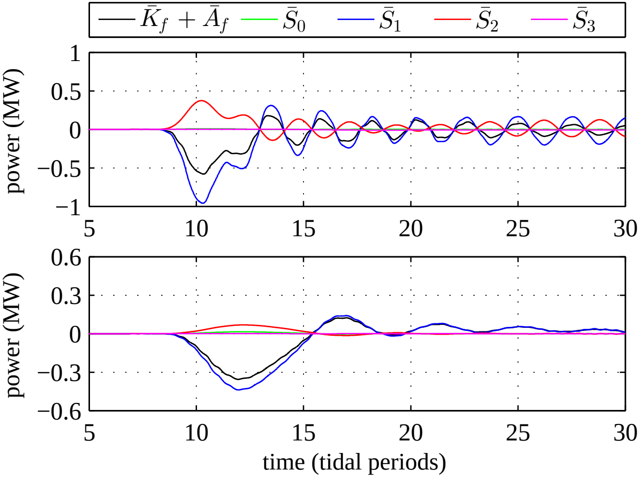


- The energy budget is computed inside the dashed circle

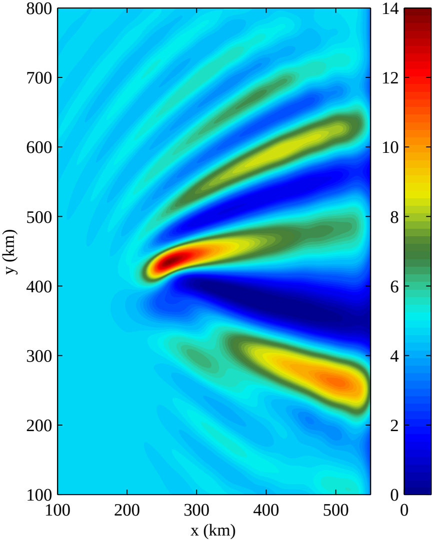


Also we have the tidal average operator,

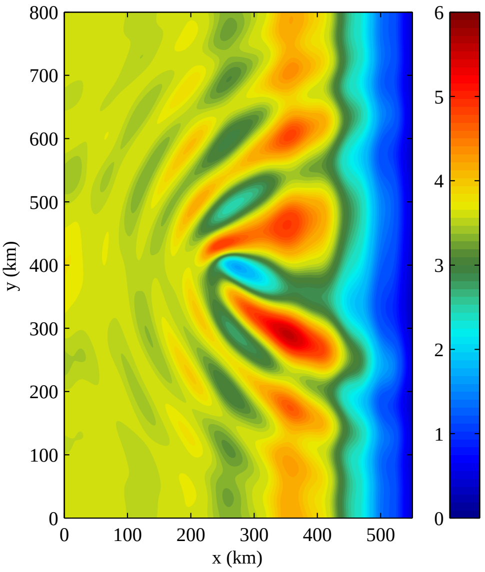
$$\bar{X}(t) = \frac{1}{T}\int_{t-T}^{t} X(t)\,\mathrm{d}t, \quad t \geq T,$$



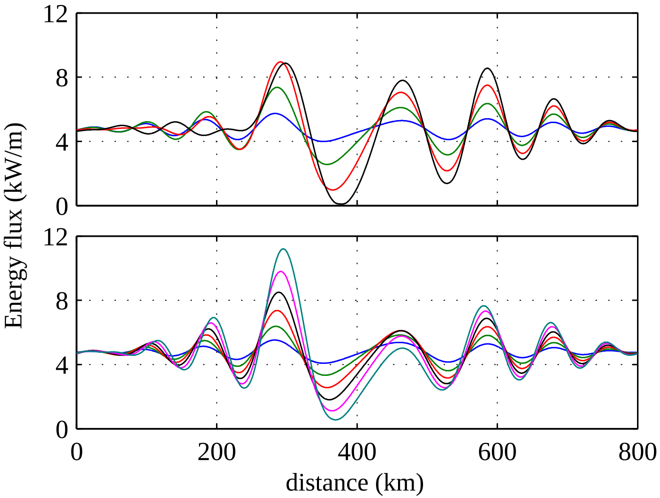
- Top: low latitude - PSI
- Bottom: mid latitude - no PSI



- Energy flux magnitude
- $|p_1' \vec{u_1}|$
- $f = 0.5 \times 10^{-4}$ s^{-1}
- $L_E = 50$ km
- $U_\theta = 45$ cm/s.
- Base flux = 4.78 kW/m.
- Average $15T < t < 16T$

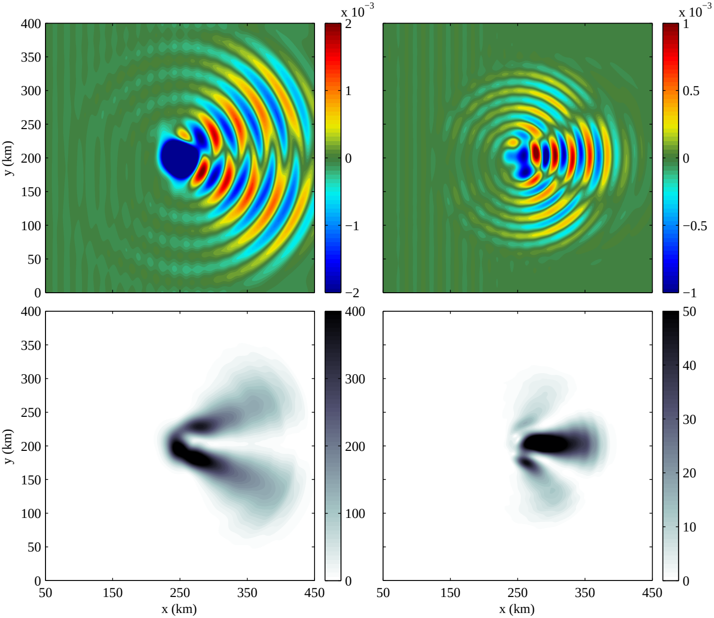


- Energy flux magnitude
- $|p_1' \vec{u_1}|$
- $f = 1.0 \times 10^{-4}\ s^{-1}$
- $L_E = 30$ km
- $U_\theta = 30$ cm/s.
- Base flux = 3.68 kW/m.
- Average $16T < t < 17T$



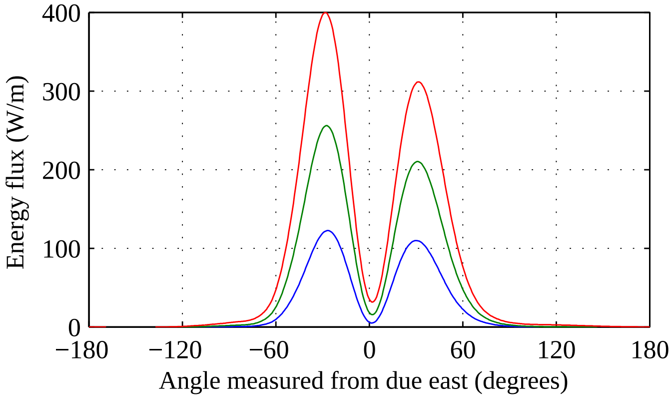
- Top: $U_\theta = 45$ cm/s, $L_E = 20, 30, 40, 50$ km
- Bottom: $L_E = 30$ km, $U_\theta = 15, 30, 45, 60, 75, 90$ cm/s.

Baroclinic eddy

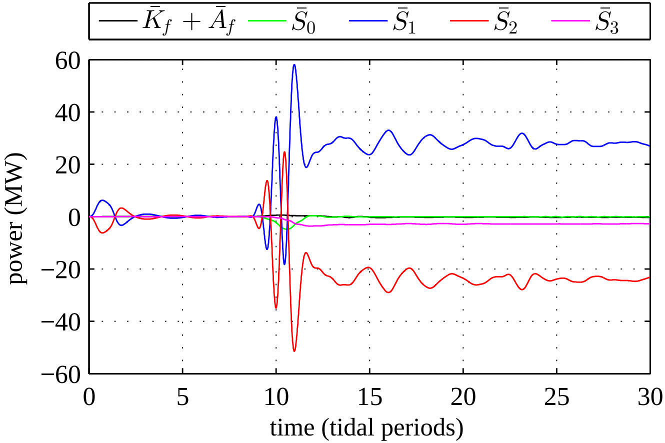


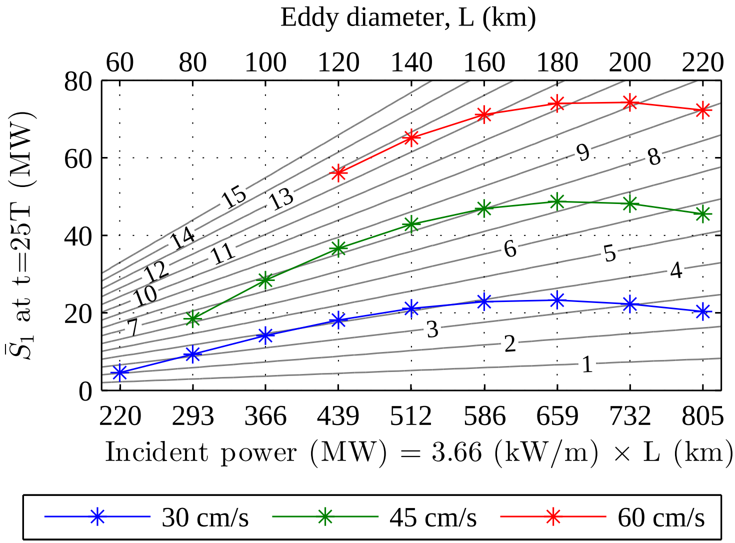
- Low f
- $L_E = 35$ km
- $U_\theta = 45$ cm/s
- Average $15T < t < 16T$
- Energy flux magnitude
- $|p_2' \vec{u_2}|$
- $|p_3' \vec{u_3}|$

- Energy flux normal to 80km radius circle
- $L_E = 35$ km, $U_\theta = 30, 45, 60$ cm/s
- Average $15T < t < 16T$

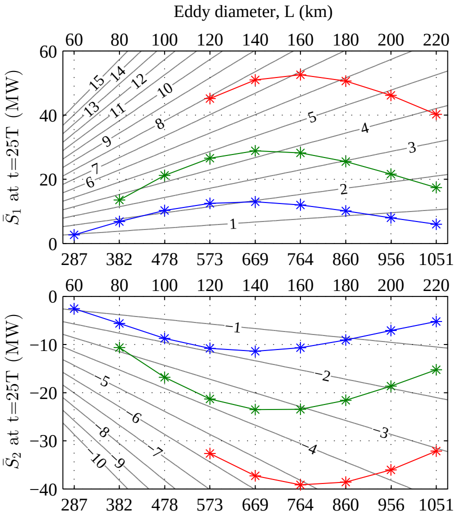


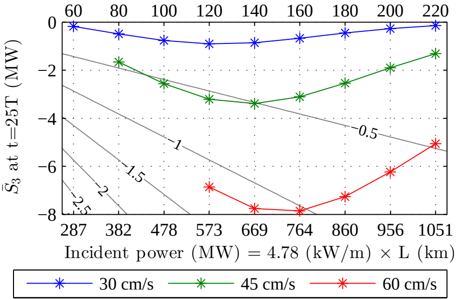
- Tidal-averaged terms for low f, $L_E = 35$ km, $U_\theta = 45$ cm/s



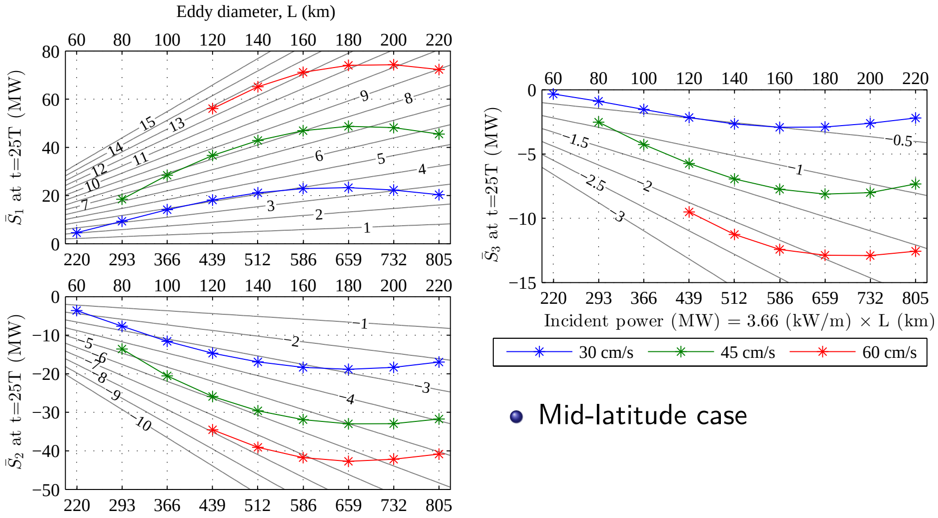

Eddy diameter, L (km)
60
80
100
120
140
160
180
200
220
80
60
40
20
0
$\bar{S}_1$ at t=25T (MW)
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
220
293
366
439
512
586
659
732
805
Incident power (MW) = 3.66 (kW/m) × L (km)
30 cm/s
45 cm/s
60 cm/s

L_E \ U_θ	30 cm/s	45 cm/s	60 cm/s
15 km	X		
20 km	X	X	
25 km	X	X	X
30 km	X	X	X
35 km	X	X	X
40 km	X	X	X
45 km	X	X	X
50 km	X	X	X
55 km	X	X	X





- Low-latitude case



- Mid-latitude case

- Potential weaknesses:

- Potential weaknesses:
- The mode-one energy budget includes the eddy and the forced mode-one wave

- Potential weaknesses:
- The mode-one energy budget includes the eddy and the forced mode-one wave
- Energy may be lost from the eddy

- Potential weaknesses:
- The mode-one energy budget includes the eddy and the forced mode-one wave
- Energy may be lost from the eddy
- However there is no evidence to support this (everything indicates resonance)

Results from this work indicate that barotropic eddies:

Results from this work indicate that barotropic eddies:

- strongly affect energy flux patterns by creating hot and cold spots of energy flux

Results from this work indicate that barotropic eddies:

- strongly affect energy flux patterns by creating hot and cold spots of energy flux
- use the constructive/destructive interference mechanism, which reduces the coherence of mode-one internal tides

Results from this work indicate that barotropic eddies:

- strongly affect energy flux patterns by creating hot and cold spots of energy flux
- use the constructive/destructive interference mechanism, which reduces the coherence of mode-one internal tides
- are not efficient at scattering energy between internal tide modes

Implications for the background field:

Results from this work indicate that barotropic eddies:

- strongly affect energy flux patterns by creating hot and cold spots of energy flux
- use the constructive/destructive interference mechanism, which reduces the coherence of mode-one internal tides
- are not efficient at scattering energy between internal tide modes

Implications for the background field:

- Stronger energy cascade in hotspots, weaker in cold spots

Results from this work indicate that baroclinic eddies:

Results from this work indicate that baroclinic eddies:

- are efficient at scattering energy to higher internal tide modes

Results from this work indicate that baroclinic eddies:

- are efficient at scattering energy to higher internal tide modes
- use the resonant triad mechanism to scatter energy

Results from this work indicate that baroclinic eddies:

- are efficient at scattering energy to higher internal tide modes
- use the resonant triad mechanism to scatter energy
- act as a drag on a mode-one internal tide
  (analogous to the topographic drag on the barotropic tide)

Results from this work indicate that baroclinic eddies:

- are efficient at scattering energy to higher internal tide modes
- use the resonant triad mechanism to scatter energy
- act as a drag on a mode-one internal tide
  (analogous to the topographic drag on the barotropic tide)

Further,

- Low mode internal tides and mesoscale eddies are highly scale-compatible for interaction

Results from this work indicate that baroclinic eddies:

- are efficient at scattering energy to higher internal tide modes
- use the resonant triad mechanism to scatter energy
- act as a drag on a mode-one internal tide
  (analogous to the topographic drag on the barotropic tide)

Further,

- Low mode internal tides and mesoscale eddies are highly scale-compatible for interaction

Implications include enhanced localised dissipation:

Results from this work indicate that baroclinic eddies:

- are efficient at scattering energy to higher internal tide modes
- use the resonant triad mechanism to scatter energy
- act as a drag on a mode-one internal tide (analogous to the topographic drag on the barotropic tide)

Further,

- Low mode internal tides and mesoscale eddies are highly scale-compatible for interaction

Implications include enhanced localised dissipation:

- Energy is shifted from mode-one to mode-two and higher

Results from this work indicate that baroclinic eddies:

- are efficient at scattering energy to higher internal tide modes
- use the resonant triad mechanism to scatter energy
- act as a drag on a mode-one internal tide
  (analogous to the topographic drag on the barotropic tide)

Further,

- Low mode internal tides and mesoscale eddies are highly scale-compatible for interaction

Implications include enhanced localised dissipation:

- Energy is shifted from mode-one to mode-two and higher
- Higher modes propagate slower, subject to more interactions

Results from this work indicate that baroclinic eddies:

- are efficient at scattering energy to higher internal tide modes
- use the resonant triad mechanism to scatter energy
- act as a drag on a mode-one internal tide
  (analogous to the topographic drag on the barotropic tide)

Further,

- Low mode internal tides and mesoscale eddies are highly scale-compatible for interaction

Implications include enhanced localised dissipation:

- Energy is shifted from mode-one to mode-two and higher
- Higher modes propagate slower, subject to more interactions
- Reduces the mode-one energy that reaches a shoreline

Future work

- Extend this work to non-constant N
- Parameterisation?

Future work

- Extend this work to non-constant N
- Parameterisation?

Related questions

- Can mode-two internal tides be observed emanating from an eddy?
  (satellite measurements, moorings, etc)

References I

P. Bartello.
Geostrophic Adjustment and Inverse Cascades in Rotating Stratified Turbulence.
Journal of Atmospheric Sciences, 52:4410–4428, December 1995.

T. H. Bell.
Topographically generated internal waves in the open ocean.
Journal of Geophysical Research, 80:320–327, January 1975. doi: 10.1029/JC080i003p00320.

References II

Oliver Bühler and Michael E. McIntyre.
Wave capture and wave-vortex duality.
Journal of Fluid Mechanics, 534:67–95, 6 2005.
ISSN 1469-7645.
doi: 10.1017/S0022112005004374.
URL http://journals.cambridge.org/article_S0022112005004374.

S. Legg and K. M. H. Huijts.
Preliminary simulations of internal waves and mixing generated by finite amplitude tidal flow over isolated topography.
Deep Sea Research Part II: Topical Studies in Oceanography, 53:140–156, January 2006.
doi: 10.1016/j.dsr2.2005.09.014.

References III

M.-P. Lelong and J. J. Riley.
Internal wave-vortical mode interactions in strongly stratified flows.
Journal of Fluid Mechanics, 232:1–19, 1991.
doi: 10.1017/S0022112091003609.