# Strain-driven submesoscale frontogenesis : what can surface currents tell us about what is happening below?

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Work funded by NSF and NOAA

# Outline

Submesoscale frontogenesis 101

Observations

Theory

Conclusions

# Characteristics of oceanic submesoscale processes

- Dynamic definition: $R_o = \zeta/f = \mathcal{O}(1)$ where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$
  $\Rightarrow$ ageostrophic
- Horizontal length scale: $\zeta = U/L \Rightarrow L \sim U/f$
  $U \sim 0.1$ m s$^{-1}$ and $f \sim 10^{-4}$ s$^{-1}$ $\Rightarrow L \sim 1$ km
- Time scale: $T \sim L/U \Rightarrow T \sim f^{-1}$ inertial period
- Vertical length scale: $H \sim$ mixed layer depth
- Vertical velocity scale: $W \sim UH/L$
  $\Rightarrow W \sim 10^{-3}$ m s$^{-1}$ $\sim 100$ m day$^{-1}$

# Why should we care about submesoscale processes?

- Strong vertical velocities ($\sim 100$ m day$^{-1}$)
  $\Rightarrow$ large nutrient supply for primary production (Levy et al., 2001; Lapeyre and Klein, 2006)
  $\Rightarrow$ large vertical fluxes of buoyancy $\Rightarrow$ Mixed Layer restratification (Lapeyre et al., 2006; Boccaletti et al., 2007)
- Submesoscale frontogenesis and instabilities cascade mesoscale kinetic energy to smaller scales (Capet et al., 2008b; Klein et al., 2008)
  $\Rightarrow$ direct energy pathway from mesoscales to mixing and dissipation

# Frontogenesis

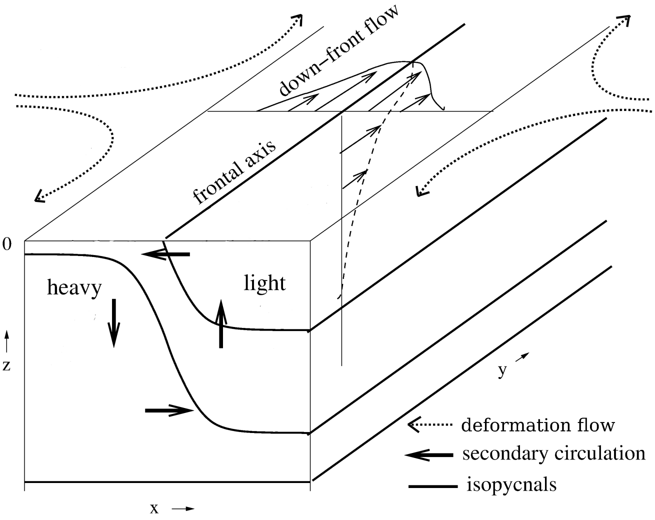



Figure: adapted from Capet et al. (2008a)

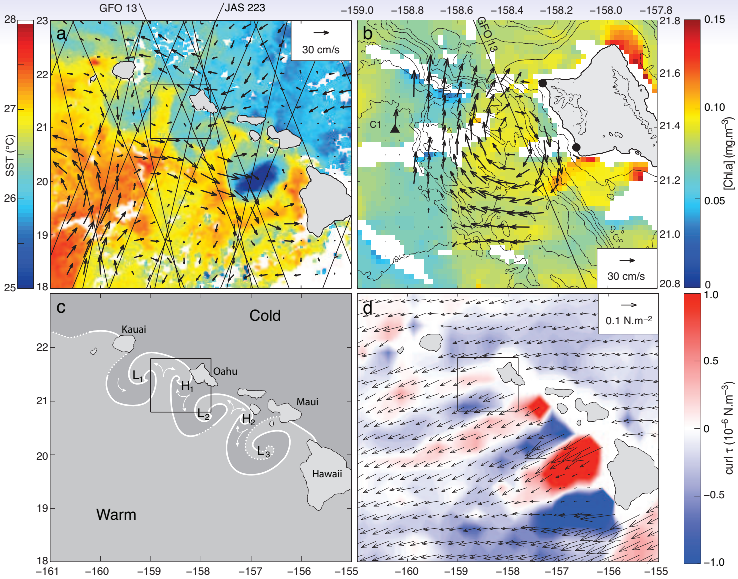
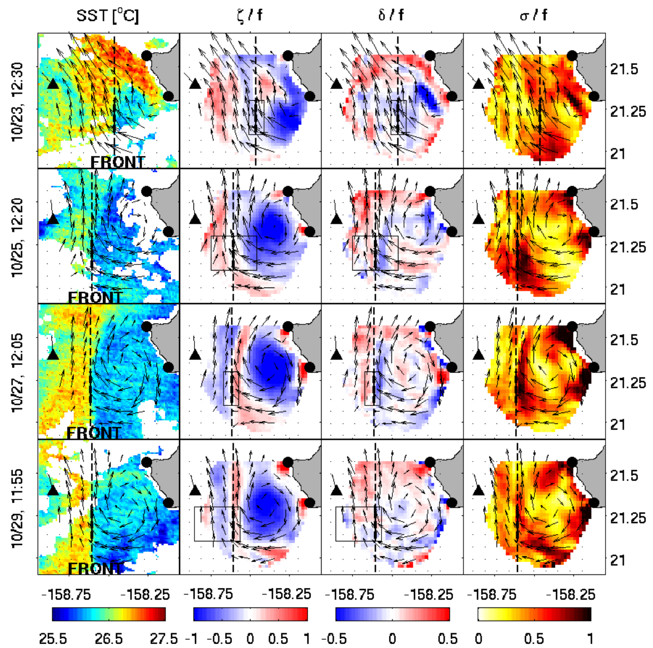



Figure: Chavanne et al. (2010)


SST [°C]
ζ / f
δ / f
σ / f
10/23, 12:30
10/25, 12:20
10/27, 12:05
10/29, 11:55
FRONT
21.5
21.25
21
-158.75
-158.25
25.5
26.5
27.5
-1
-0.5
0
0.5
1

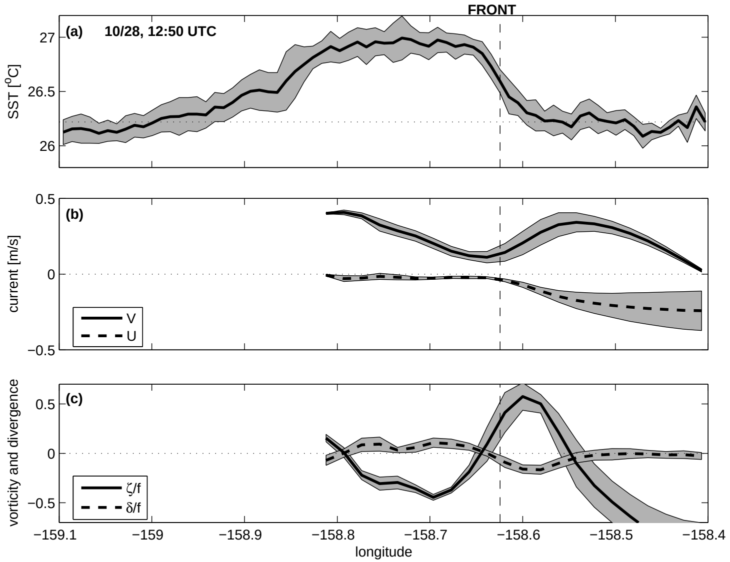



Figure: Chavanne et al. (2010)

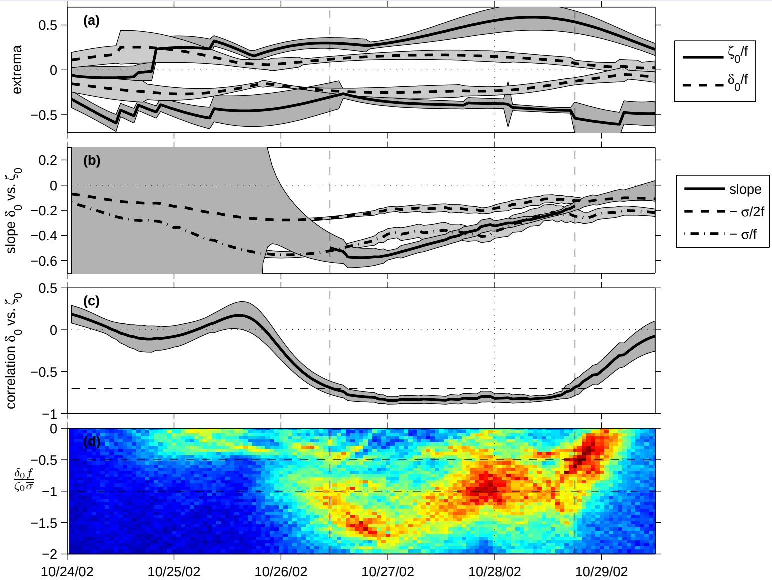



Figure: Chavanne et al. (2010)

# Straight front

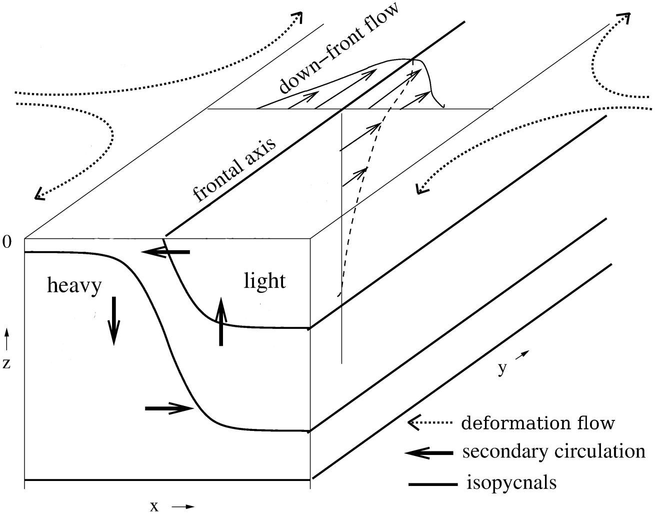



Figure: adapted from Capet et al. (2008a)

# Flow decomposition

Following Hoskins & Bretherton (1972), let us decompose the flow into a barotropic confluent non-divergent background flow and the flow associated with a straight front aligned in the y-direction, assumed independent of y:

$$u_{tot} = -\frac{\bar{\sigma}(t)}{2}x + u_a(x,z,t) \quad (1)$$

$$v_{tot} = \frac{\bar{\sigma}(t)}{2}y + v_g(x,z,t) \quad (2)$$

$$w_{tot} = w(x,z,t) \quad (3)$$

$$\phi_{tot} = \Phi(x,y,z,t) + \phi(x,z,t) \quad (4)$$

$$b_{tot} = B(z) + b(x,z,t) \quad (5)$$

where $\bar{\sigma}(t)$ is the background strain rate.

# Geostrophic coordinates

Hoskins & Bretherton (1972) introduced the geostrophic coordinates:

$$X = x + \frac{v_g}{f} \quad (6)$$

$$Z = z \quad (7)$$

$$T = t \quad (8)$$

The Jacobian of the coordinates transformation is:

$$J = 1 + \frac{1}{f}\frac{\partial v_g}{\partial x} = \frac{1}{1 - \frac{1}{f}\frac{\partial v_g}{\partial X}} \quad (9)$$

# Equations of motion

The inviscid equations of motion on the $f$-plane in the semi-geostrophic approximation and geostrophic coordinates are:

$$fv_g = \frac{\partial \psi}{\partial X} \quad (10)$$

$$\frac{Dv_g}{DT} + \frac{\bar{\sigma}}{2} v_g + f u_a^* = 0 \quad (11)$$

$$b = \frac{\partial \psi}{\partial Z} \quad (12)$$

$$\frac{Db}{DT} + \frac{q}{f} w^* = 0 \quad (13)$$

$$\frac{\partial u_a^*}{\partial X} + \frac{\partial w^*}{\partial Z} = 0 \quad (14)$$

# Equations of motion

where:

$$
\begin{aligned}
\psi &= \phi + \frac{v_g^2}{2} && (15)\\
\frac{D}{DT} &= \frac{\partial}{\partial T} - \frac{\bar{\sigma}}{2} X \frac{\partial}{\partial X} && (16)\\
u_a^* &= u_a + \frac{1}{f} w \frac{\partial v_g}{\partial Z} && (17)\\
w^* &= \frac{w}{J} && (18)\\
q &= fJ \frac{\partial (b + B)}{\partial Z} \approx fN^{*2} + f\left( \frac{\partial^2 \psi}{\partial Z^2} + \frac{N^{*2}}{f^2} \frac{\partial^2 \psi}{\partial X^2} \right) && (19)\\
N^{*2} &= \frac{\partial B}{\partial Z} && (20)
\end{aligned}
$$

and q is the potential vorticity.

# Equations of motion

The thermal wind balance is:

$$f \frac{\partial v_g}{\partial Z} = \frac{\partial b}{\partial X} \tag{21}$$

The vertical velocity is governed by the omega equation:

$$\frac{1}{f} \frac{\partial^2 q w^*}{\partial X^2} + f^2 \frac{\partial^2 w^*}{\partial Z^2} = \bar{\sigma} \frac{\partial^2 b}{\partial X^2} = f \bar{\sigma} \frac{\partial \zeta^*}{\partial Z} \tag{22}$$

where $\zeta^* = \frac{\partial v_g}{\partial X}$.

# Davies & Müller (1988)

Vertical domain: $-\infty < Z \leq 0$

Boundary conditions: $w = 0$ at $Z = 0$ and $w \to 0$ as $Z \to -\infty$

Uniform background stratification: $N^{*2}$ = constant

Uniform potential vorticity: $\frac{\partial^2 \psi}{\partial Z^2} + \frac{N^{*2}}{f^2} \frac{\partial^2 \psi}{\partial X^2} = 0$

$\Rightarrow v_g = \hat{v}_0 e^{iKX} e^{\frac{N^* K}{f} Z}$

Solution to omega equation: $\hat{w}^* = \frac{\bar{\sigma} \hat{\zeta}_0^*}{2f} Z e^{\frac{N^* K}{f} Z}$

$\Rightarrow \delta_0 = -\frac{\bar{\sigma}}{2f} \zeta_0$

Non-dimensional parameter: $\kappa = \frac{\delta_0 f}{\zeta_0 \bar{\sigma}} = -\frac{1}{2}$

# Hoskins & Bretherton (1972)

Vertical domain: $-H \leq Z \leq 0$

Boundary conditions: $w = 0$ at $Z = \{-H, 0\}$

Zero potential vorticity: $\frac{\partial b}{\partial Z} = -\frac{\partial B}{\partial Z}$

$\Rightarrow \frac{\partial^2 b}{\partial X \partial Z} = 0 \Rightarrow v_g$ is linear in $Z$

Conservation of mass $\Rightarrow v^g(Z = -H) = -v^g(Z = 0)$

$\Rightarrow \frac{\partial \zeta^*}{\partial Z} = \frac{2\zeta_0^*}{H}$

Solution to omega equation: $\hat{w}^* = \frac{\bar{\sigma}}{fH}\zeta_0^* Z(Z + H)$

$\Rightarrow \delta_0 = -\frac{\bar{\sigma}}{f}\zeta_0$

Non-dimensional parameter: $\kappa = \frac{\delta_0 f}{\zeta_0 \bar{\sigma}} = -1$

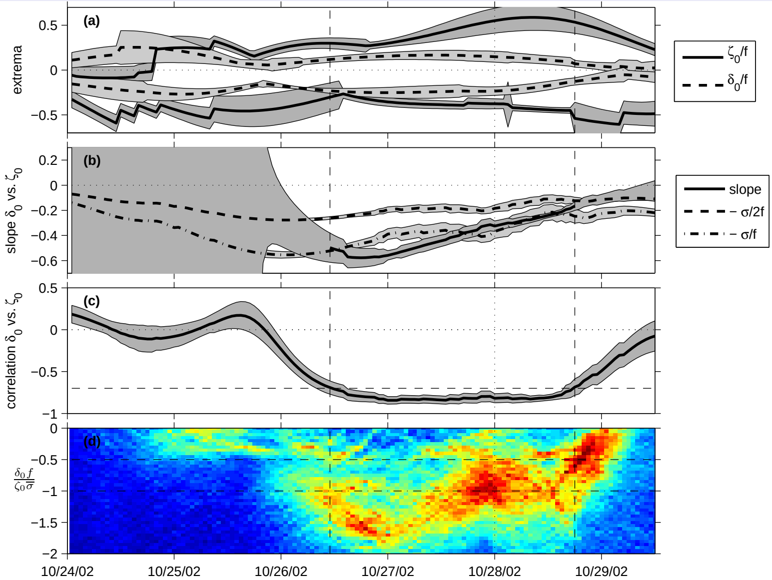



Figure: Chavanne et al. (2010)

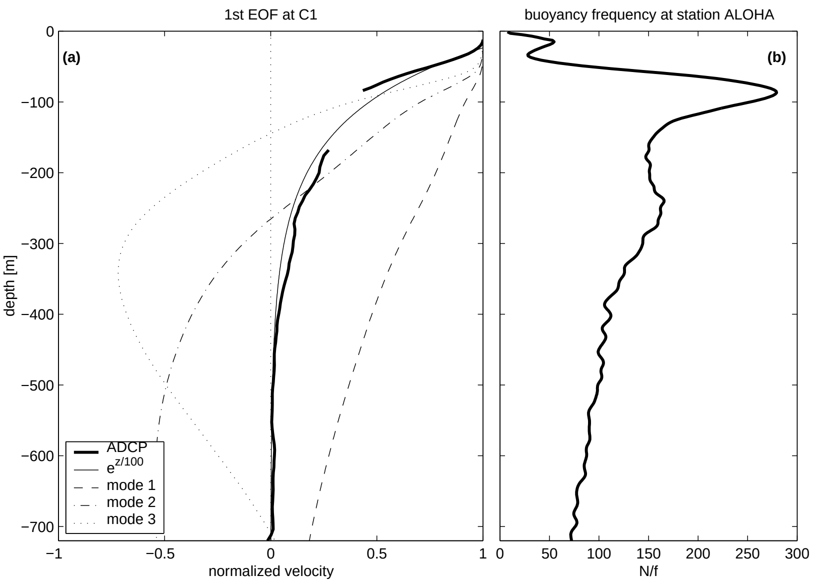



Figure: Chavanne et al. (2010)

# Conclusions

- Observations of a submesoscale frontogenesis event in Hawaii are quantitatively explained by the semi-geostrophic zero-PV finite-layer model of Hoskins and Bretherton (1972).
- The semi-geostrophic constant-PV semi-infinite layer model of Davies & Müller (1988) only qualitatively explains the observations.
- This suggests that the front was confined to the surface mixed-layer and decoupled from the ocean interior by a strong pycnocline, as indicated by "nearby" hydrographic observations.

Boccaletti, G., R. Ferrari, and B. Fox-Kemper (2007). Mixed layer instabilities and restratification. *J. Phys. Oceanogr.* 37:22282250.

Capet X, McWilliams JC, Molemaker M, Shchepetkin A. (2008a). Mesoscale to submesoscale transition in the California current system. Part II: Frontal processes. *J. Phys. Oceanogr.* 38:4464.

Capet X, McWilliams JC, Molemaker M, Shchepetkin A. (2008b). Mesoscale to submesoscale transition in the California current system. Part III: Energy balance and flux. *J. Phys. Oceanogr.* 38:2256-2269.

Chavanne, C., P. Flament, and K.W. Gurgel (2010). Interactions between a submesoscale anticyclonic vortex and a front. *J. Phys. Oceanogr.* 40:18021818.

Davies, H. C., and J. C. Müller (1988). Detailed description of deformation-induced semi-geostrophic frontogenesis. *Quart. J. Roy. Meteor. Soc.* 114:12011219.

Hoskins, B. J., and F. P. Bretherton (1972). Atmospheric frontogenesis models: Mathematical formulation and solution. *J. Atmos. Sci.* 29:1137.

Klein P, Hua BL, Lapeyre G, Capet X, LeGentil S, Sasaki H. (2008). Upper ocean turbulence from high 3-D resolution simulations. *J. Phys. Oceanogr.* 38:174863.

Lapeyre G, Klein P. (2006). Impact of the small-scale elongated filaments on the oceanic vertical pump. *J. Mar. Res.* 64:83551.

Lapeyre G, Klein P, Hua BL. (2006). Oceanic restratification by surface frontogenesis. *J. Phys. Oceanogr.* 36:157790.

Lévy M, Klein P, Tréguier AM. (2001). Impact of submesoscale physics on production and subduction of phytoplankton in an oligotrophic regime. *J. Mar. Res.* 59:53565.