Modeling the schistosomiasis on the islets in Nanjing

Longxing Qi

School of Mathematical Sciences, Anhui University

LAMPS and Department of Mathematics and Statistics, York University

Background: Patients, Schistosome, Snail

Background

- Schistosomiasis

Background

- Schistosomiasis
  - One of the most prevalent parasitic diseases

Background

- Schistosomiasis
  - One of the most prevalent parasitic diseases
  - 207 million people

Background

- Schistosomiasis
  - One of the most prevalent parasitic diseases
  - 207 million people
  - Difficult to eradicate

Background

- Schistosomiasis
  - One of the most prevalent parasitic diseases
  - 207 million people
  - Difficult to eradicate
- Four factors in transmission:

Background

- Schistosomiasis
  - One of the most prevalent parasitic diseases
  - 207 million people
  - Difficult to eradicate
- Four factors in transmission:
  - Definitive hosts—-human, mammals

Background

- Schistosomiasis
  - One of the most prevalent parasitic diseases
  - 207 million people
  - Difficult to eradicate
- Four factors in transmission:
  - Definitive hosts—-human, mammals
  - Intermediate hosts—-snails

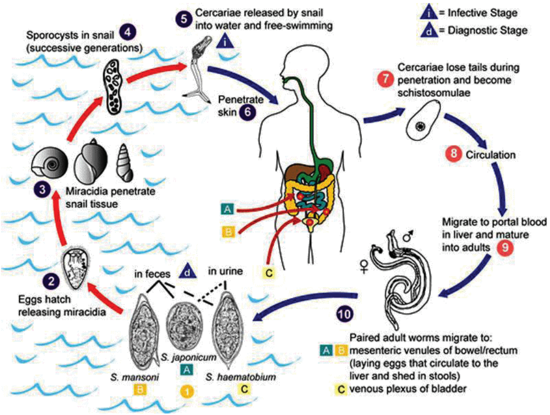
Background

- Schistosomiasis
  - One of the most prevalent parasitic diseases
  - 207 million people
  - Difficult to eradicate
- Four factors in transmission:
  - Definitive hosts—-human, mammals
  - Intermediate hosts—-snails
  - Schistosome

Background

- Schistosomiasis
  - One of the most prevalent parasitic diseases
  - 207 million people
  - Difficult to eradicate
- Four factors in transmission:
  - Definitive hosts—-human, mammals
  - Intermediate hosts—-snails
  - Schistosome
  - Water

Life cycle of schistosome



Two islets in Nanjing

Two islets in Nanjing

- Two islets:

Two islets in Nanjing

- Two islets:
  - Qianzhou, Zimuzhou, in the center of the Yangtze River near Nanjing

Two islets in Nanjing

- Two islets:
  - Qianzhou, Zimuzhou, in the center of the Yangtze River near Nanjing
  - No human residents or other livestocks

Two islets in Nanjing

- Two islets:
  - Qianzhou, Zimuzhou, in the center of the Yangtze River near Nanjing
  - No human residents or other livestocks
  - *Rattus norvegicus* infected by schistosome

Two islets in Nanjing

- Two islets:
  - Qianzhou, Zimuzhou, in the center of the Yangtze River near Nanjing
  - No human residents or other livestocks
  - *Rattus norvegicus* infected by schistosome
  - Snails

Two islets in Nanjing

- Two islets:
  - Qianzhou, Zimuzhou, in the center of the Yangtze River near Nanjing
  - No human residents or other livestocks
  - *Rattus norvegicus* infected by schistosome
  - Snails
- What will happen ?

Two islets in Nanjing

- Two islets:
  - Qianzhou, Zimuzhou, in the center of the Yangtze River near Nanjing
  - No human residents or other livestocks
  - *Rattus norvegicus* infected by schistosome
  - Snails
- What will happen ?
- How to control schistosomiasis on this two islets ?

Model

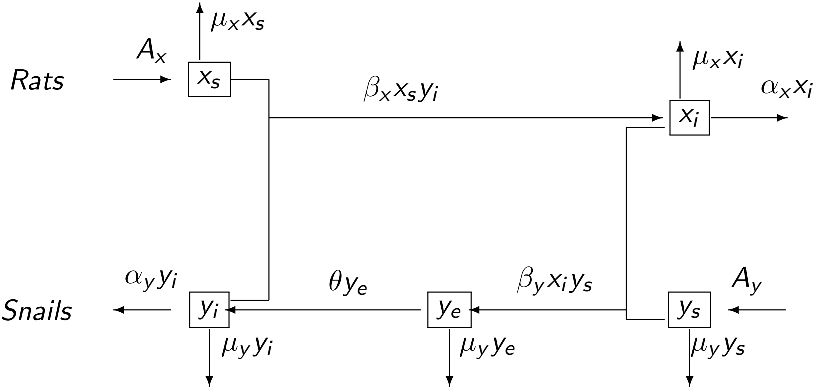


Figure: The flow diagram of schistosomiasis activities.

Model

$$\begin{cases} \dfrac{dx_s}{dt} = A_x - \mu_x x_s - \beta_x x_s y_i, \\ \dfrac{dx_i}{dt} = \beta_x x_s y_i - (\mu_x + \alpha_x) x_i, \\ \dfrac{dy_s}{dt} = A_y - \mu_y y_s - \beta_y x_i y_s, \\ \dfrac{dy_e}{dt} = \beta_y x_i y_s - (\mu_y + \theta) y_e, \\ \dfrac{dy_i}{dt} = \theta y_e - (\mu_y + \alpha_y) y_i. \end{cases} \tag{1}$$

Model

$$
\begin{cases}
\dfrac{dx_s}{dt} = A_x - \mu_x x_s - \beta_x x_s y_i, \\
\dfrac{dx_i}{dt} = \beta_x x_s y_i - (\mu_x + \alpha_x) x_i, \\
\dfrac{dy_s}{dt} = A_y - \mu_y y_s - \beta_y x_i y_s, \\
\dfrac{dy_e}{dt} = \beta_y x_i y_s - (\mu_y + \theta) y_e, \\
\dfrac{dy_i}{dt} = \theta y_e - (\mu_y + \alpha_y) y_i.
\end{cases} \tag{1}
$$

- $\frac{A_x}{\mu_x}$, $\frac{A_y}{\mu_y}$: the density—-closely related to the area of the two islets.
- Chunhua Shan and Professor Huaiping Zhu: The Dynamics of Growing Islets and Transmission of Schistosomiasis Japonica in the Yangtze River (To appear in *Bulletin of Mathematical Biology*)

Parameters

- A_x, per capita reproduction rate of rats;
- μ_x, per capita natural death rate of rats;
- α_x, per capita disease-induced death rate of rats;
- β_x, per capita contact transmission rate from infected snails to susceptible rats;
- A_y, per capita reproduction rate of snails;
- μ_y, per capita natural death rate of snails;
- α_y, per capita disease-induced death rate of snails;
- β_y, per capita contact transmission rate from infected rats to susceptible snails;
- θ, per capita transition rate from infected and preshedding snails to shedding snails.

Existence of equilibria

Existence of equilibria

The basic reproduction number:

$$R_0 = \rho(FV^{-1}) = \sqrt[3]{\frac{A_x A_y \theta \beta_x \beta_y}{\mu_x \mu_y (\mu_x + \alpha_x)(\mu_y + \alpha_y)(\mu_y + \theta)}}.$$

Existence of equilibria

The basic reproduction number:

$$R_0 = \rho(FV^{-1}) = \sqrt[3]{\frac{A_x A_y \theta \beta_x \beta_y}{\mu_x \mu_y (\mu_x + \alpha_x)(\mu_y + \alpha_y)(\mu_y + \theta)}}.$$

- The disease free equilibrium $E_0 = (\frac{A_x}{\mu_x}, 0, \frac{A_y}{\mu_y}, 0, 0)$,
- The unique endemic equilibrium $E^* = (x_s^*, x_i^*, y_s^*, y_e^*, y_i^*) \Leftarrow R_0 > 1$

Stability of equilibria

Using a Lyapunov function:

$$\begin{aligned} V &= \beta_y y_s^* x_i^* \{[x_s - x_s^* - x_s^* \ln(\frac{x_s}{x_s^*})] + [x_i - x_i^* - x_i^* \ln(\frac{x_i}{x_i^*})]\} \\ &\quad + \beta_x x_s^* y_i^* \{[y_s - y_s^* - y_s^* \ln(\frac{y_s}{y_s^*})] + [y_e - y_e^* - y_e^* \ln(\frac{y_e}{y_e^*})] \\ &\quad + \frac{\mu_y + \theta}{\theta}[y_i - y_i^* - y_i^* \ln(\frac{y_i}{y_i^*})]\}, \end{aligned}$$

and by LaSalle's Invariance Principle, the stability is

Theorem

The disease free equilibrium E_0 of the system (1) is globally asymptotically stable if $R_0 \leq 1$.

Theorem

For system (1), if $R_0 > 1$, the endemic equilibrium E^* is globally asymptotically stable.

Data

The data are based on the investigation of Nanjing Institute of Parasitic Diseases in the period of 1996-1998.

Table: Dissection results of snails from Qianzhou and Zimuzhou islets in 1996-1998

Islet	Year	No.dissected	No.positive (%)
Qianzhou	1996	2677	53 (2.0)
	1997	8205	53 (0.6)
	1998	7538	234(3.1)
Zimuzhou	1997	6324	25 (0.4)
	1998	5440	27 (0.5)

Data

Table: Dissection results of rats from Qianzhou and Zimuzhou islets in 1996-1998

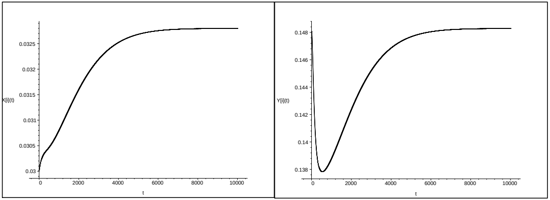
Islet	Year	No.dissected	No.positive (%)
Qianzhou	1996.12-1997.3	69	43 (62.3)
	1997.12-1998.3	53	34 (64.2)
Zimuzhou	1997.12-1998.3	67	36 (53.7)
Total		189	113 (59.8)

Parameter estimation

parameters	values(per capita per day)	references
A_x	0.00006	estimated;
μ_x	9.13×10^{-4}	Xugy,1999
α_x	8.33×10^{-5}	Ishikawa,2006
β_x	0.007	estimated;
A_y	0.108	estimated;
μ_y	2.63×10^{-3}	Anderson,1992, Feng,2005
α_y	4.67×10^{-3}	Feng,2005 and Zhou,1988
β_y	0.0009	estimated
θ	2.5×10^{-2}	Allen,2003

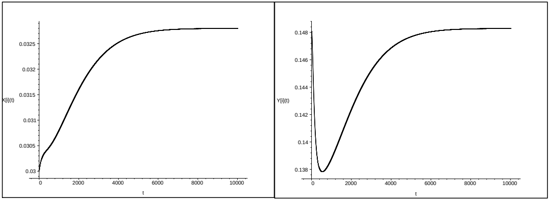
Simulation

Figure: The trajectories of x_i and y_i



Simulation

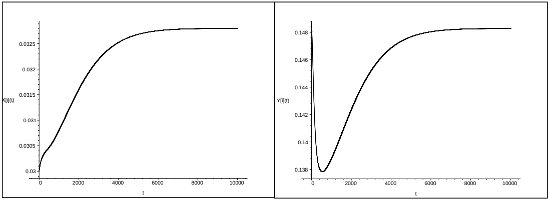
Figure: The trajectories of x_i and y_i



- $R_0 = 1.29 > 1$,

Simulation

Figure: The trajectories of x_i and y_i



- $R_0 = 1.29 > 1$,
- Schistosomiasis will be prevalent on this two islets.

Control measures

- Rats:

Control measures

- Rats:
  - Mousetraps

Control measures

- Rats:
  - Mousetraps
  - The rate of capture rats: 0.16–0.35/day

Control measures

- Rats:
  - Mousetraps
  - The rate of capture rats: 0.16–0.35/day
  - Expensive

Control measures

- Rats:
  - Mousetraps
  - The rate of capture rats: 0.16−0.35/day
  - Expensive
- Snails:

Control measures

- Rats:
  - Mousetraps
  - The rate of capture rats: 0.16−0.35/day
  - Expensive
- Snails:
  - Molluscicides

Control measures

- Rats:
    - Mousetraps
    - The rate of capture rats: 0.16−0.35/day
    - Expensive
- Snails:
    - Molluscicides
    - The death rate of snails: 0.8−1/day

Control measures

- Rats:
  - Mousetraps
  - The rate of capture rats: 0.16−0.35/day
  - Expensive
- Snails:
  - Molluscicides
  - The death rate of snails: 0.8−1/day
  - Cheap

Control measures

- Rats:
  - Mousetraps
  - The rate of capture rats: 0.16−0.35/day
  - Expensive
- Snails:
  - Molluscicides
  - The death rate of snails: 0.8−1/day
  - Cheap
- Which one should we choose if only one of them can be chosen?

Control measures

- Rats:
  - Mousetraps
  - The rate of capture rats: 0.16−0.35/day
  - Expensive
  - k_x be the rate of control rats per day,
- Snails:
  - Molluscicides
  - The death rate of snails: 0.8−1/day
  - Cheap
- Which one should we choose if only one of them can be chosen?

Control measures

- Rats:
  - Mousetraps
  - The rate of capture rats: 0.16−0.35/day
  - Expensive
  - k_x be the rate of control rats per day,
- Snails:
  - Molluscicides
  - The death rate of snails: 0.8−1/day
  - Cheap
  - k_y be the rate of control snails per day.
- Which one should we choose if only one of them can be chosen?

Control model

Control model

$$
\begin{cases}
\dfrac{dx_s}{dt} = A_x - (\mu_x + k_x)x_s - \beta_x x_s y_i, \\
\dfrac{dx_i}{dt} = \beta_x x_s y_i - (\mu_x + k_x + \alpha_x)x_i, \\
\dfrac{dy_s}{dt} = A_y - (\mu_y + k_y)y_s - \beta_y x_i y_s, \\
\dfrac{dy_e}{dt} = \beta_y x_i y_s - (\mu_y + k_y + \theta)y_e, \\
\dfrac{dy_i}{dt} = \theta y_e - (\mu_y + k_y + \alpha_y)y_i.
\end{cases}
\tag{2}
$$

Control model

$$\begin{cases} \dfrac{dx_s}{dt} = A_x - (\mu_x + k_x)x_s - \beta_x x_s y_i, \\ \dfrac{dx_i}{dt} = \beta_x x_s y_i - (\mu_x + k_x + \alpha_x)x_i, \\ \dfrac{dy_s}{dt} = A_y - (\mu_y + k_y)y_s - \beta_y x_i y_s, \\ \dfrac{dy_e}{dt} = \beta_y x_i y_s - (\mu_y + k_y + \theta)y_e, \\ \dfrac{dy_i}{dt} = \theta y_e - (\mu_y + k_y + \alpha_y)y_i. \end{cases} \tag{2}$$

The basic reproduction number for model (2):

$$R_0^* = \sqrt[3]{\frac{A_x A_y \theta \beta_x \beta_y}{(\mu_x + k_x)(\mu_y + k_y)(\mu_x + k_x + \alpha_x)(\mu_y + k_y + \alpha_y)(\mu_y + k_y + \theta)}}.$$

Sensitivity analysis

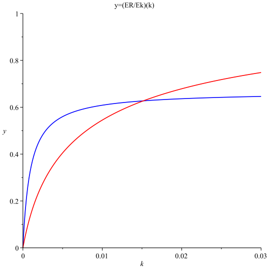
The flexibility of R_0^* to k_x and k_y are given by:

$$\frac{ER_0^*}{Ek_x} = -\frac{1}{3}k_x(\frac{1}{\mu_x + k_x} + \frac{1}{\mu_x + k_x + \alpha_x}),$$

$$\frac{ER_0^*}{Ek_y} = -\frac{1}{3}k_y(\frac{1}{\mu_y + k_y} + \frac{1}{\mu_y + k_y + \alpha_y} + \frac{1}{\mu_y + k_y + \theta}).$$

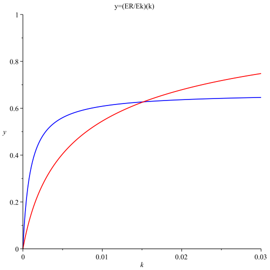
Sensitivity analysis

Figure: The plotting of $|\frac{ER_0^*}{Ek}|$ over k_x and k_y



Sensitivity analysis

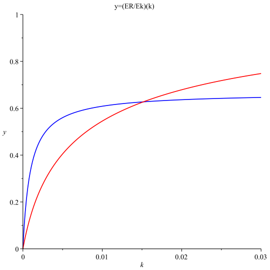
Figure: The plotting of $|\frac{ER_0^*}{Ek}|$ over k_x and k_y



- The blue curve: $|\frac{ER_0^*}{Ek_x}|$ over k_x,

Sensitivity analysis

Figure: The plotting of $|\frac{ER_0^*}{Ek}|$ over k_x and k_y



- The blue curve: $|\frac{ER_0^*}{Ek_x}|$ over k_x,
- The red curve: $|\frac{ER_0^*}{Ek_y}|$ over k_y,

Sensitivity analysis

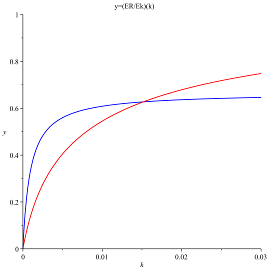


Figure: The plotting of $|\frac{ER_0^*}{Ek}|$ over k_x and k_y

- The blue curve: $|\frac{ER_0^*}{Ek_x}|$ over k_x,
- The red curve: $|\frac{ER_0^*}{Ek_y}|$ over k_y,
- The intersection point: $k_y = 0.019$.

Sensitivity analysis

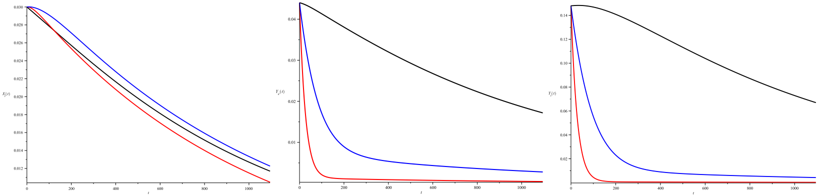
- R_0^* is more sensitive to k_y than to k_x when $k_y > 0.019$,

Sensitivity analysis

- R_0^* is more sensitive to k_y than to k_x when $k_y > 0.019$,
- To control snails is easier to eliminate schistosomiasis than to control rats as long as $k_y > 0.019$.

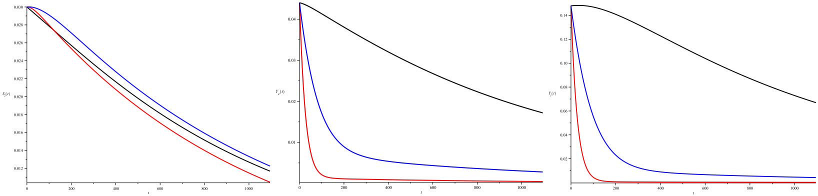
Compare two control measures

Figure: The trajectories of x_i, y_e and y_i



Compare two control measures

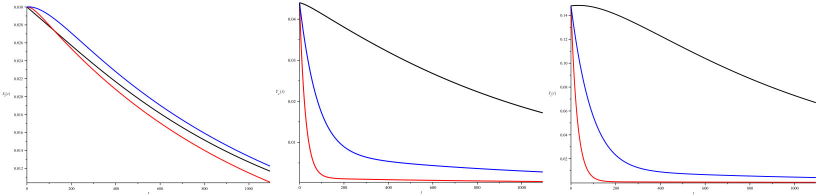
Figure: The trajectories of x_i, y_e and y_i



- The black curve: $k_x = 0.001$ and $k_y = 0$,

Compare two control measures

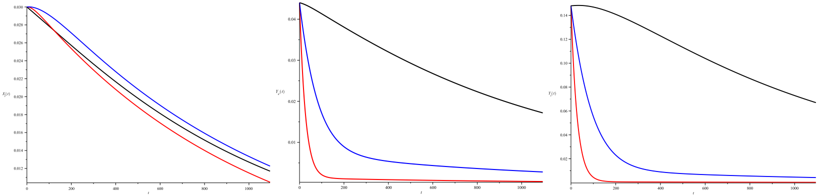
Figure: The trajectories of x_i, y_e and y_i



- The black curve: $k_x = 0.001$ and $k_y = 0$,
- The blue curve: $k_x = 0$ and $k_y = 0.01 < 0.019$,

Compare two control measures

Figure: The trajectories of x_i, y_e and y_i



- The black curve: $k_x = 0.001$ and $k_y = 0$,
- The blue curve: $k_x = 0$ and $k_y = 0.01 < 0.019$,
- The red curve: $k_x = 0$ and $k_y = 0.036 > 0.019$.

Conclusion

If only one of control measures can be chosen

Conclusion

If only one of control measures can be chosen

- To control snails is more efficient than to control rats,

Conclusion

If only one of control measures can be chosen

- To control snails is more efficient than to control rats,
- Make sure the rate of controlling snails be greater than 0.019.

Thank you!