Notes on the illumination parameters
of convex polytopes

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Let $\mathbf{K}$ be a convex body of $\mathbf{E}^d$. We call a point $\ell \in \mathbf{E}^d \setminus \mathbf{K}$ a light-source and say that it illuminates the boundary point p of $\mathbf{K}$ if the half-line starting at ℓ and passing through p intersects the interior of $\mathbf{K}$ somewhere not between ℓ and p. The set of light-sources $\{\ell_1, \ell_2, \ldots, \ell_n\} \subset \mathbf{E}^d \setminus \mathbf{K}$ illuminates $\mathbf{K}$ if each boundary point of $\mathbf{K}$ is illuminated by at least one of the light-sources. If $\mathbf{K}$ is symmetric about the origin then the *illumination parameter* of $\mathbf{K}$ is defined as

$$IP(\mathbf{K}) = \inf \left\{ \sum_i \|\ell_i\|_{\mathbf{K}} : \{\ell_i\} \text{ illuminates } \mathbf{K} \right\},$$

where $\|x\|_{\mathbf{K}} = \inf\{0 < \lambda : \lambda^{-1} x \in \mathbf{K}\}$. (This ensures that far-away light-sources are penalised.)

Convex bodies with large illumination parameters are constructed in each dimension. The exact values of the illumination parameters of the centrally symmetric Platonic solids are calculated, and estimates on the illumination parameters of the centrally symmetric Archimedean solids are given.