ZDENEK DVORAK
Charles University

Deciding first-order properties for sparse graphs

For a fixed graph H, testing whether there exists a homomorphism to H is typically NP-complete. On the other hand, testing whether there exists a homomorphism from H is trivially solvable in polynomial time. However, the exponent of the polynomial depends on H, and since the problem is $W[1]$-hard, it is unlikely that the dependence could be eliminated. However, by the result of Eppstein, the problem becomes FPT for planar graphs. We present a generalization of this result in two directions: instead of the homomorphism problem, we consider first-order properties, and instead of planar graphs, we allow an arbitrary class with (locally) bounded expansion. This strengthens all previous resuls on testing first-order properties on classes of sparse graphs. The results are based on the joint work with D. Král' and R. Thomas.

JAN FONIOK
École Polytechnique (ParisTech)

Adjoint functors on digraphs

I will show a number of old and new applications of adjoint functors on categories of relational structures. They include a construction of CSPs of width 1 (a.k.a. tree duality); a construction of graphs with a high chromatic number and no short odd cycles; resource-sharing system scheduling; and the failure of several potential approaches to Hedetniemi's product-coloring conjecture.

HAMED HATAMI
McGill University

Elementary asymptotic extremal graph theory is non-trivial

In this talk, we introduce two kinds of power for graphs. First, for a given graph G, we consider $G^{\frac{r}{s}}$, i.e., the rth power of the sth subdivision of G, and we present some basic properties of this power. In the sequel, we introduce the graph power $G^{\frac{2s+1}{2r+1}}$. We show that these powers can be considered as the dual of each other. Precisely, the purpose of this talk is to show that even the most elementary problems in asymptotic extremal graph theory can be highly non-trivial. We study linear inequalities between graph homomorphism densities. It is known that every such inequality follows from an infinite number of certain applications of the Cauchy-Schwarz inequality. Lovasz and, in a slightly different formulation, Razborov asked whether it is true or not that every algebraic inequality

between the homomorphism densities follows from a "finite" number of applications of the Cauchy-Schwarz inequality. In this talk, we show that the answer to this question is negative by showing that the problem of determining the validity of a linear inequality between homomorphism densities is undecidable.

This talk is based on a joint work with Sergey Norin.

PAVOL HELL
Simon Fraser University

Homomorphism Dichotomies and Graph Classes

I will discuss recent results classifying the complexity of list homomorphism and of minimum cost homomorphism problems, by certain combinatorial conditions. These conditions in turn suggest interesting classes of graphs and especially of digraphs, analogous to well studied graph classes such as interval and unit interval graphs. These results are joint with Arash Rafiey, Tomas Feder, and Huang Jing.

DANIEL KRAL
Charles University

Algorithmic metatheorems

Algorithmic metatheorems guarantee that certain types of problems have efficient algorithms. A classical example is the theorem of Courcelle asserting that every MSOL property can be tested in linear time for graphs with bounded tree-width. In this talk, we survey existing results on testing MSOL and FOL properties for various classes of combinatorial structures, graphs and matroids in particular, and present their relation to graph homomorpisms. Among others we mention our recent result with Tomas Gavenciak and Sang-il Oum on matroids with locally bounded branch-width and our result with Zdenek Dvorak and Robin Thomas on testing FOL properties in graphs with (locally) bounded expansion. The latter result will be discussed in more detail in the talk of Zdenek Dvorak.

BENOIT LAROSE
Champlain Regional College

The Complexity of the List Homomorphism Problem for Graphs

We present a complete classification of the computational complexity of the list H-colouring problem for graphs (with possible loops) in combinatorial and algebraic terms: for every graph H, the problem is either NP-complete, NL-complete, L-complete or is first-order definable; descriptive complexity equivalents are given as well via Datalog and its fragments. Our algebraic characterisations match important conjectures in the study of constraint satisfaction problems. Joint work with L. Egri, A. Krokhin and P. Tesson.

JAROSLAV NESETRIL OR PATRICE OSSONA DE MENDEZ

The existence and counting of subgraphs

We survey algorithmic and asymptotic results related to the title. Particularly, for sparse classes we interpret the asymptotic value of these parameters as degree of freedom. This in turn characterizes the somewhere dense vs nowhere dense dichotomy of graph classes.

JAROSLAV NESETRIL OR PATRICE OSSONA DE MENDEZ

On First-Order definable H-colorings

We address the following problem: what classes C admit first-order definable H-colorings, that is H-colorings problems equivalent *in the class* to the satisfaction of a formula expressed in first-order logic. We relate the existence of such first-order definable coloring problems to the concept of classes of bounded expansion, and propose a conjecture linked to a weakening of a conjecture by Erdos and Hajnal and to a conjecture due to Thomassen.

ARASH RAFIEY HAFSHEJANI
IDSIA

Minimum Cost Homomorphism Problem for Digraphs

We consider an optimization version of the homomorphism problem that was motivated by a real-world problem in defence logistics. For each vertex of input digraph D there is a cost (non-negative number) of mapping that vertex to a vertex of target digraph H. The minimum cost homomorphism problem, MinHOM(H), seeks a homomorphism f from D to H with minimum total cost.

We give a classification of digraphs H for which MinHOM(H) is polynomial. Our classification is combinatorial and testable in polynomial time.

The list homomorphism problem for H, LHOM(H), asks whether an input digraph D admits a homomorphism to H subject to restrictions (lists) of allowed images for each vertex of D. Clearly, LHOM(H) is reducible to MinHOM(H). The dichotomy for LHOM(H) was previously proved by Bulatov.

If time allows I will also mention our combinatorial classification of digraphs H for which LHOM(H) problem is tractable. These are joint results with Pavol Hell.

MARK SIGGERS
Kyungpook National University

Semi-lattice Polymorphisms on Reflexive graphs

Near-unanimity polymorphisms on reflexive graphs have many nice characterizations. Semilattice polymorphisms, another main weak near-unanimity polymorphisms, is less studied.

We define several types of semi-lattice polymorphisms on reflexive graphs based on how the edges of the graph are situated in relation to semi-lattice ordering.

Considering the classes of reflexive graphs that admit semi-lattice polymorphisms with various combinations of these properties, we get a hierarchy of graph classes that in a natural way generalize chordal graphs.

We look at various aspects of this hierarchy with the goal of characterizing both the classes of reflexive graphs admitting semi-lattice polymorphisms of various types, and the intersection of these classes with the classes of graphs admitting near-unanimity polymorphisms.

CLAUDE TARDIF
Royal Military College, Kingston

Duality and Exponential graphs

Some questions about colourings and homomorphisms of categorical products of graphs can be approached by proving the existence of homomorphisms from exponential graphs into target graphs.

I will present two specific instances:

- El-Zahar and Sauer's proof that the chromatic number of the product of two 4-chromatic graphs is 4.

- The density of the lattice of powers of a fixed complete graph in the category of directed graphs.

In both cases, the proof relies on homomorphisms whose existence is not established constructively but rather dually, by proving the nonexistence of homomorphism from obstructions to the domain. This requires a suitable characterisation of the obstructions to a structure with bounded width.

PETER WINKLER
Dartmouth College

Random Walks and Other Homomorphisms

A simple random walk on a graph G may be thought of as a random homomorphism from a path to G. We'll review some of the (many) pretty results about random walk and discuss the possibility of extending them to random homomorphisms from other graphs.

XUDING ZHU
National Sun Yat-sen University

Circular colouring of graphs

Given a real number r and a graph G, a circular r-colouring of G colours vertices of G with points in a circle of circumference r so that colours assigned to adjacent vertices are at least distance 1 apart. Circular colouring of graphs is a refinement of the conventional vertex colouring of graphs. The concept has atttract a lot of attention in the past twenty years, and becomes an important branch of chromatic graph theory, with many exciting results and new techniques. This talk gives a survey on some recent results in this area.