FIELDS

ABSTRACTS 1.2

AMNON BESSER
Ben-Gurion University

On the p-adic analogue of hermitian line bundles

I will explain my theory of line bundles with log functions on them, which is the p-adic analogue of hermitian line bundles. I will then explain some ideas on what one can do to extend this theory in two directions - to find an analogue of hermitian vector bundles on the one hands, and to have a good theory on the projective line, where currently the theory does not work.

ENRICO BOMBIERI
Institute for Advanced Study

Roots of polynomial in subgroups of F_p^* and applications to congruences

The study of congruences $(\bmod p)$ for sparse polynomials in one variable arises in several contexts from diophantine problems to computer science. Usual methods based on Fourier analysis $(\bmod p)$ do not work well when differences between exponents have very large common factors with $p-1$. In this work, jointly with J. Bourgain and V.S. Konyagin, we introduce a new technique to handle the difficult case. The main new tool shows in a precise quantitative way that it is not possible to describe a 'large' subgroup of F_p^* by means of equations of 'small' degree and height; this is yet another example of the principle of 'independence' between addition and multiplication in a finite field. The reduction of the original problem about congruences to the principle alluded to above involves a study of the geometry of intersections of varieties of Fermat type, the Arithmetic Bézout Theorem, the Arithmetic Hilbert Nullstellensatz, as well as certain higher dimensional variants of the above principle of independence. The arguments involved are elementary but not entirely obvious, in the authors' opinion.

JEAN-BENOîT BOST
Université Paris XI – Orsay

Lefschetz theorems on arithmetic schemes

Classical Lefschetz theorems compare the topology of a projective variety and of its hyperplane sections. Their modern avatars in SGA2 deal with vector bundles over a scheme and their formal completion along a closed subscheme. This talk will be devoted to arithmetic counterparts of the theorems in SGA2, concerning arithmetic surfaces and arithmetic threefolds, and to some Diophantine applications.

JOSE IGNACIO BURGOS
Barcelona

The height of toric subvarieties

This is a report on joint work with P. Philippon and M. Sombra. A complete toric variety X of dimension n is determined by a lattice N and a complete integral fan Σ in $N_{\mathbb{R}}$. This variety has a model over the integers and has an action of a torus T. A equivariant ample line bundle L on X determines an integral polytope P in $M_{\mathbb{R}} = N_{\mathbb{R}}^{\vee}$. The algebro-geometric properties of (X, L) can easily be read from the polytope P. The exponential map determines a parametrization of the principal open subset X_0 (the maximal orbit under the action of T) by $N_{\mathbb{C}}$. Assume that L is provided with a hermitian metric that is equivariant under the action of the compact torus. Then, minus the logarithm of the norm of a section of L, determines a function f on $N_{\mathbb{R}}$. If the hermitian metric is positive, this function is strictly convex. Its stability set is the polytope P. Then the Legendre dual $g = f^{\vee}$ is a strictly convex function on P. This is the symplectic potential in the Guillemin-Abreu theory. We prove that the height of X with respect to $\overline{L}$ is given by $(n+1)!$ times the integral of $-g$ with respect to a measure of P. This is the arithmetic analogue of the expression of the degree of a toric variety as $n!$ times the volume of the polytope.

We expect that many other Arakelov theory properties of X can be read from the function g.

ANTOINE CHAMBERT-LOIR
IRMAR

Rationality of formal functions on arithmetic surfaces

I will discuss a rationality criterion for formal functions along a divisor on an arithmetic surface which generalizes the classical rationality theorems of Borel, Dwork, Plya, Bertrandias for power series. The proof uses diophantine approximation techniques (via slope estimates) to prove algebraicity, capacity theory at all places for algebraic curves and the Hodge index theorem in Arakelov geometry. This is joint work with Jean-Benot Bost.

J-L. COLLIOT-THÉLÈNE
CNRS, Université Paris-Sud

Obstruction de Brauer-Manin entière pour les espaces homogènes

Une forme quadratique entière peut être représentée par une autre forme quadratique entière sur tous les anneaux d'entiers p-adiques et sur les réels, sans l'être sur les entiers. On en trouve de nombreux exemples dans la littérature. Nous montrons qu'une partie de ces exemples s'explique au moyen d'une obstruction de type Brauer-Manin pour les points entiers. Pour plusieurs types d'espaces homogènes de groupes algébriques linéaires, cette obstruction est la seule obstruction à l'existence d'un point entier. On décrit un travail de Fei Xu et l'orateur (espaces homogènes d'un groupe semsimple) ainsi qu'un travail de D. Harari (espaces homogènes d'un tore).

Integral Brauer-Manin obstruction for homogeneous spaces

An integer may be represented by a quadratic form over each ring of p-adic integers and over the reals without being represented by this quadratic form over the integers. More generally, such failure of a local-global principle may occur for the representation of one integral quadratic form by another integral quadratic form. Many such examples may be accounted for by a Brauer-Manin obstruction for the existence of integral points on schemes defined over the integers. For several types of homogeneous spaces of linear algebraic groups, this obstruction is shown to be the only obstruction to the existence of integral points. This talk describes work of Fei Xu and the speaker (homogeneous spaces of semisimple groups) and work of Harari (homogeneous spaces of a torus).

CATERINA CONSANI
Johns Hopkins

On the notion of geometry over F_1

In the talk I will review and refine the notion of variety (of finite type) over the "field with one element" as developed by C. Soule and show that this new notion becomes compatible with the geometric viewpoint originally developed by J. Tits. In the second part of the talk I will apply this construction to show that Chevalley schemes are examples of varieties defined over a quadratic extension of the above "field". This is a joint work with A. Connes.

PIETRO CORVAJA
Udine

Integral points, divisibility between values of polynomials and entire curves on surfaces

I shall report on a series of joint works with Umberto Zannier, concerning integral points on surfaces. We recently worked out some concrete applications to rational surfaces; in particular, we connect the distribution of integral points on such surfaces with divisibility between values of polynomials in two variables at integral points. All our results admit a counterpart in complex geometry, asserting the algebraic degeneracy of entire curves on certain surfaces.

JAN-HENDRIK EVERTSE
University of Leiden

On the Quantitative Subspace Theorem

In 1989, Schmidt obtained a quantitative version of his Subspace Theorem, giving an explicit upper bound for the number of subspaces of the Diophantine inequality involved. His result was generalized and improved in various directions. The best and most general result obtained so far was obtained in 2002 by Schlickewei and the author. In fact, they obtained a quantitative version of the Subspace Theorem over number fields.

Like all the previous quantitative versions of the Subspace Theorem, the proof of Schlickewei and the author was basically a quantification of Schmidt's original argument, which was based on an extension of Roth's Lemma and geometry of numbers. In 1994, Faltings and Wüstholz gave a totally different proof of the Subspace Theorem, based on Faltings' Product Theorem. Also the method of Faltings and Wüstholz can be used to obtain an explicit upper bound for the number of subspaces, but it leads to a much larger bound.

In the present lecture I present a further improvement of the quantitative Subspace Theorem, obtained jointly with Roberto Ferretti, which is obtained by combining ideas from Schmidt's method of proof with that of Faltings and Wüstholz.

KÁLMÁN GYORY
University of Debrecen

S-unit equations in number fields: effective results, generalizations, applications, abc conjecture

The S-unit equations have many important applications and generalizations. In our talk we first present some explicit bounds, obtained jointly with K. Yu, for the solutions of S-unit equations in two unknowns in algebraic number fields. Recently these bounds were used by the speaker to derive the best known effective estimates in the direction of the uniform *abc* conjecture over number fields. We then formulate a recent effective finiteness theorem, due to Pintér and the speaker, for a common generalization of S-unit equations and binomial Thue-equations with unknown exponents. This provides a general effective result for the corresponding three-parameter family of S-unit equations. Finally, another generalization will be discussed for polynomial equations in two unknowns with algebraic coefficients. In joint work with Bérczes, Evertse and Pontreau we have recently derived explicit upper bounds for the solutions of such equations, when the unknowns are taken from algebraic numbers that, with respect to the height, are "close" to a given multiplicative subgroup of finite rank of $\overline{\mathbb{Q}}^*$. The main feature of these results is that the solutions under consideration do not have to lie in a prescribed number field. Hence we gave bounds not only for the heights but also for the degrees of the solutions. These results of ours generalize a theorem of Bombieri and Gubler. Further, for the special class of varieties we consider, they can be regarded as effective versions of some very general but ineffective theorems of Laurent, Poonen, Evertse, Schlickewei, Schmidt and Rémond, respectively.

EUGENE HA
Johns Hopkins

On vector bundles and the adele-class space

A noncommutative-geometric framework for the Riemann-Weil explicit formula for Hecke L-functions has been developed by Connes, with further improvements by Connes-Consani-Marcolli and R. Meyer. At the core of this approach is the study of the adele-class space of a global field K, which is the quotient of the adeles of K by the multiplicative action of the units of K. In our talk we shall restrict ourselves to the case where K is the rational number field and discuss a geometric model of the adele-class space based on the geometry of the Bost-Connes system, Durov's notion of Z-infinity, and vector bundles on an Arakelov compactification of Spec(Z).

PHILIPP HABEGGER
ETH Zurich

Height Upper Bounds on Abelian Varieties and Algebraic Tori

We describe results on height upper bounds related to conjectures stated by Bombieri, Masser, Zannier and Pink and Zilber in the context of abelian varieties and algebraic tori. More specifically, given a 'sufficiently general' r-dimensional subvariety of such a group variety one expects a point outside a natural non-dense expectational locus which is contained in an algebraic subgroup of codimension at least r to have uniformly bounded height. A general approach is based on a geometry of numbers type argument together with analytic methods revolving around a theorem of Ax. Another line of attack, at least in the toric case, uses ideas from tropical geometry. We also discuss some finiteness and non-density results which are consequences of the height bound and of other results in diophantine geometry such as Lehmer-type inequalities.

NORIKO HIRATA-KOHNO
Nihon University

Unit equations having few solutions

Let K be a field of characteristic 0. We consider unit equations in unknowns in a multiplicative group of finite rank in K^n. We introduce an equivalence among unit equations such that all equations from one class have the same number of solutions. It is well-known that every unit equation has only finitely many solutions, which is due to C. L. Siegel in number field case and A. van der Poorten - H. P. Schlickewei in general case. Improved related results are shown by J.-H. Evertse - Schlickewei - W. M. Schmidt in 2002. On the other hand, Evertse -K. Győry proved in 1988 in general case that all but finitely many equivalence classes, unit equations have few solutions and the set of solutions is contained in no more than $2^{(n+1)!}$ proper linear subspaces. Later, Evertse improved the upper bound of the number to 2^n; that of proper linear subspaces containing the set of non-degenerate solutions. Here we give a bound of the number of exceptional equivalence classes and also a refined estimate of the number of proper linear subspaces in some situation.

MINHYONG KIM
University College London

Selmer varieties

This lecture will be a progress report on a program for studying the Diophantine geometry of hyperbolic curves using non-abelian cohomology varieties associated to arithmetic fundamental groups.

KLAUS KÜNNEMAN
Universität Regensburg

Line bundles with connections on projective varieties over function fields and number fields

We report about joint work with with Jean-Benoit Bost (Orsay). Consider a hermitian line bundle on a smooth, projective variety over a number field. The arithmetic Atiyah class of the hermitian line vanishes by definition if and only if the unitary connection on the hermitian line bundle is already defined over the number field. We show that this can happen only if the class of the line bundle is torsion. This problem may be translated into a concrete problem of diophantine geometry, concerning rational points of the universal vector extension of the Picard variety. We investigate this problem, which was already considered and solved in some cases by Bertrand, by using a classical transcendence result of Schneider-Lang. We also consider a geometric analog of our arithmetic situation, namely a smooth, projective variety which is fibered on a curve defined over some field of characteristic zero. To any line bundle on the variety is attached its Atiyah class relative to the base curve. We describe precisely when this relative class vanishes. In particular, when the fixed part of the relative Picard variety is trivial, this holds only when the restriction of the line bundle to the generic fiber of the fibration is a torsion line bundle.

AARON LEVIN
Scuola Normale Superiore

Runge's method and the effective computation of integral points

We discuss some variations on the classical method of Runge for effectively computing integral points on certain curves. In particular, we will expand the class of curves to which Runge's method can be effectively applied. As an application, we will discuss the solutions of some particular Diophantine equations (e.g., (almost) squares in arithmetic progressions).

GAËL RÉMOND
Institut Fourier

Heights of Jacobians and rational points

We present an upper bound for the theta height of the Jacobian of a curve in terms of a projective height of the curve. We apply this to the problem of finding an upper bound for the number of rational points on the curve (assumed to be of genus at least two) over a number field. A previous result gave a formula involving the said theta height and the Mordell-Weil rank of the Jacobian. Here we derive an explicit upper bound in terms of more naive data from the curve and the field.

CHRISTOPHE SOULÉ
CNRS and IHES

Linear projections and successive minima

Let X be an arithmetic surface and L a line bundle on X. Choose a metric h on the lattice Λ of sections of L over X. When the degree of the generic fiber of X is large enough, we get lower bounds for the successive minima of (Λ, h) in terms of the normalized height of X. The proof uses an effective version (due to C. Voisin) of a theorem of Segre on linear projections, and Morrison's proof that smooth projective curves of high degree are Chow semi-stable.

LUCIEN SZPIRO
CUNY

Algebraic Dynamics

YU YASUFUKU
CUNY

On Vojta's Conjecture

Vojta's conjecture is a powerful conjecture in Diophantine geometry, implying among other things equivalences between sporadicity of rational points and various notions of hyperbolicity. In this talk, we will discuss and prove some cases of Vojta's conjecuture, including cases of blowup varieties and cases of dynamic interests. We will also mention their arithmetic consequences, such as to greatest common divisors and gaps of S-units. A major ingredient is Schmidt's subspace theorem, both directly and via the gcd inequality of Corvaja and Zannier.

SHOU-WU ZHANG
Columbia University

Gross-Schoen cycles, dualising sheaves, and tautological classes

In this talk, I will state an identity between the height of the Gross–Schoen cycle on the triple product of a curve and the self-intersection of the relative dualising sheaf. I will discuss some consequences on the Gillet–Soule conjecture, Bogomolov conjecture, Beilinson–Bloch conjecture, and Beauville's tautological classes in the Jacobian.