ARITHMETIC AND GEOMETRY OF ALGEBRAIC VARIETIES
WITH SPECIAL EMPHASIS ON
CALABI–YAU VARIETIES AND MIRROR SYMMETRY
MARCH 15–16, 2008

ABSTRACTS

Ethan Cotterill (Queen's University)

Geometry of curves with exceptional secant planes

We study curves with linear series that are exceptional with regard to their secant planes. Working in the framework of an extension of Brill–Noether theory to pairs of linear series, we prove that a general curve of genus g has no exceptional secant planes, in a very precise sense. We also address the problem of computing the number of linear series with exceptional secant planes in a one-parameter family in terms of tautological classes associated with the family. We obtain conjectural generating functions for the tautological coefficients of secant-plane formulas associated to series g^{2d-1}_m that admit d-secant $(d-2)$-planes. We also describe a strategy for computing the classes of divisors associated to exceptional secant plane behavior in the Picard group of the moduli space of curves in a couple of naturally-arising infinite families of cases, and we give a formula for the number of linear series with exceptional secant planes on a general curve equipped with a one-dimensional family of linear series.

Snigdhayan Mahanta (The Fields Institute/University of Toronto)

Noncommutative geometry and stability conditions

I shall present a model of noncommutative geometry based on differential graded categories generalising the approach based on triangulated categories after Keller, Toen et al. Bridgeland introduced a space of stability conditions on triangulated categories motivated by Π-stability in physics. Borrowing physical ideas I shall discuss why the space of stability conditions is believed to be closely related to the Kaehler moduli space. However, this should not be taken as the final answer.

Eddy Lee (UCLA)

A quotient of the Schoen quintic

The Schoen quintic in P^4 was one of the first examples of a modular Calabi-Yau threefold. I will go over its construction and geometric properties, construct a quotient which is also modular, and discuss some related issues.

James Lewis (University of Alberta)

Residues of algebraic cycles

It is well-known among experts on algebraic cycles that Beilinson's formulation of the Hodge conjecture for the higher K-groups of smooth complex quasiprojective varieties is false. We will present an amended version of this conjecture, and show how it relates to other conjectures due to Jannsen, Voisin and Bloch–Kato.

Steven Lu (UQAM, Montral)

The fine structure of varieties of maximal albanese dimension

A variety whose albanese map is generically finite is called of maximal albanese dimension. We will prove a form of Lang's conjecture for general type varieties in this category, which, together with the structure theorem of Ueno, Kawamta and Viehweg, give a refined structure theory for these varieties.

Mike Roth (Queen's University)

Cup products for line bundles on complete flag varieties

Let G be a semisimple algebraic group, B a borel subgroup, and $X = G/B$. If L is a line bundle on X then by the Borel-Weil-Bott theorem there is at most one value of d for which the cohomology group $H^d(X, L)$ is nonzero. Given two line bundles L_1 and L_2 with nonzero cohomologies in degrees d_1 and d_2 it is natural to look at the cup-product map

$$H^{d_1}(X, L_1) \otimes H^{d_2}(X, L_2) \longrightarrow H^d(X, L)$$

where $d = d_1 + d_2$ and L is the tensor product $L_1 \otimes L_2$.

By the Borel-Weil-Bott theorem the cup product map is either surjective or zero, but it was not known how to tell which occurs.

This talk will give a complete answer in the A_n, B_n, C_n, and D_n cases, as well as partial answers for other semisimple G. A large part of the talk will be an exposition of the Borel-Weil-Bott theorem and some of the geometry of the varieties G/B. If there is time we will also address the representation theoretic question of which components of a tensor product of irreducible representations can be realized via cup product, and the connection of this question with the generalized Horn problem.

Matthias Schuett (Harvard University)

K3 surfaces and modular forms

A classical construction of Shimura associates every Hecke eigenform of weight 2 with rational coefficients to an elliptic curve over **Q**. The converse statement that every elliptic curve over **Q** is modular, is the Taniyama-Shimura-Weil conjecture, proven by Wiles et al.

For higher weight, however, the opposite situation applies: Nowadays we know the modularity for wide classes of varieties, but it is an open problem whether all newforms of fixed weight with rational coefficients can be realised in a single class of varieties.

I will present joint work with N. Elkies that provides the first solution to the realisation problem in higher weight: We show that every newform of weight 3 with rational coefficients is associated to a singular K3 surface over **Q**.

Jeng-Daw Yu (Queen's University)

Notes on Calabi–Yau differential equations of order four

Given a Calabi–Yau (ordinary) differential equation of order four and assume there is a corresponding pencil of Calabi–Yau varieties, we write down the (local, relative) horizontal sections of the associated (relative) de Rham cohomology near the maximally degenerate point. Using this, one can write down analytic formulas for the roots of the frobenius of generic members in the reduction of the pencil. Other observations, different approaches and the relation with the Hasse invariants will also be discussed.

Noriko Yui (Queen's University)

Arithmetic of certain K3-fibered Calabi–Yau threefolds

This is a progress report on a joint work with Y. Goto and R. Kloosterman. We first construct Calabi–Yau threefolds with K3-fibrations, and then discuss their arithmetic properties. In particular, we want to compute zeta-functions and L-series, and then discuss modularity of certain motives.

Hui June Zhu (SUNT at Buffalo)

Constructing analytic families of crystalline representations

In this talk we will construct analytic families of all irreducible crystalline representations over the absolute Galois group of K where K is a finite unramified extension of the p-adic rationals $\mathbf{Q}_p$. As an application, among other things, this allows us to prove a generalization of Breuil's conjecture on reduction types of modulo p.