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The fixed point on compacta property of topological groups

A topological group G is said to have the fixed point on compacta property if G has a fixed point whenever it acts continuously on a compact space. This is an exclusive property of 'massive' (or 'infinite dimensional') groups: no locally compact group is such. (As an exercise, prove the 1961 result by Ellis: the action of a discrete group on its Stone-Čech compactification is free.) Up until 1975, examples of topological groups with the f.p.c. property were unknown, and the first such examples made an impression of esoteric, exotic counter-examples. Beginning with the 1983 work by Gromov and Milman, who have shown that the infinite unitary groups have f.p.c., the situation has changed as lot. Nowadays such groups are known to be quite common among the 'infinite dimensional' groups, while the f.p.c. property is closely linked to geometry of large dimensions and to Ramsey theory and is used to extract information about the dynamical properties of 'massive' groups not necessarily having the f.p.c. property. We will survey all the major results, examples, and open questions of this emerging theory.