THE FIELDS INSTITUTE
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ABSTRACTS 1.2

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Derived categories of coherent sheaves on degenerations of elliptic curves

This research is in particular motivated by the recent development in homological mirror symmetry. It is well-known that an indecomposable object of the derived category of coherent sheaves on a smooth elliptic curve is isomorphic to a direct sum of shifts of vector bundles and skyscraper sheaves. Indecomposable vector bundles on elliptic curves were classified by Atiyah in 1957. An essential feature of this description is that an idecomposable vector bundle is described by two discrete parameters: rank r and degree d and *one* continuous parameter: a point of the curve. Suppose now that a family of elliptic curves degenerates into a cycle of projective lines. A natural question is: what happens with the derived category of coherent sheaves under this degeneration?

As we shall see, the derived category of coherent sheaves on a cycle of projective lines resembles the situation in the smooth case. There are three types of indecomposable objects: shifts of skyscraper sheaves at a smooth point of the curve, and the so called *bands* $\mathcal{B}(w, m, \lambda)$ and *strings* $\mathcal{S}(w)$. A band $\mathcal{B}(w, m, \lambda)$ depends on *one* continuous parameter $\lambda \in \mathbf{k}^*$, a natural number $m \in \mathbb{N}$ (which can be interpreted as a "thickening" of an object) and on a quite complicated discrete parameter w. For example, all vector bundles are bands. Torsion free sheaves, which are not vector bundles, are strings. In fact, strings are exactly indecomposable complexes from $D^-(\mathrm{Coh}_X)$ of infinite homological dimension.

In terms of our combinatoric we describe complexes corresponding to coherent sheaves and, in particular, to vector bundles, torsion free sheaves, and skyscrapersheaves. We prove various formulas describing their homological invariants. These resuls were obtained in collaboration with Yu.Drozd.

In the case of a rational curve with one simple node we also describe all stable vector bundles and semi-stable vector bundles of degree zero. We establish an explicit correspondence (given by the Fourier-Mukai transform, introduced for the singular curves by R.Friedman, J.Morgan and E.Witten) between semi-stable torsion free sheaves of degree zero and skyscraper sheaves.

Results concerning Fourier-Mukai transform were obtained in collaboration with B.Kreussler.