THE FIELDS INSTITUTE
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ABSTRACTS 1.2

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Subgroups of Finite Abelian Groups (50-60)

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Let Λ be an artin algebra. We denote by $\mathcal{S}(\Lambda)$ the category of pairs (A', A) where A is a finitely generated Λ-module and A' is a submodule of A; a map $f : (A', A) \to (B', B)$ is just a Λ- linear map $A \to B$ such that the condition $f(A') \subseteq B'$ holds. The case $\Lambda = \mathbb{Z}/p^n\mathbb{Z}$ is attracting a lot of interest since G. Birkhoff in 1934 has asked for a description of the isomorphism classes in $\mathcal{S}(\mathbb{Z}/p^n\mathbb{Z})$. In his manuscript titled "Subgroups of Abelian Groups" he has observed that in case $n = 6$ there are families of indecomposables in $\mathcal{S}(\Lambda)$. For the corresponding categories $\mathcal{S}(\Lambda)$, where Λ is the bounded polynomial ring $k[T]/T^n$, investigations concerning the cases $n \leq 6$ (finite and tame type) have been presented to the Beijing conference in 2000. One of the aims of this lecture will be to outline in which way these results can be carried over to the classical situation with $\Lambda = \mathbb{Z}/p^n\mathbb{Z}$. Also, we are going to deal with the cases $n \geq 7$. It will be shown that these cases are controlled wild with a control class consisting of a single indecomposable object. For $n \leq 6$, all the objects of $\mathcal{S}(\mathbb{Z}/p^6\mathbb{Z})$ have a τ-period dividing six. It will be shown that this is true for arbitrary n. This result is a consequence of a rather general construction in submodule categories. As a byproduct, we obtain a formula for the number of indecomposables in each of the categories $\mathcal{S}(\mathbb{Z}/p^n\mathbb{Z})$ where $1 \leq n < 6$. In addition, some related problems of dealing with sets of submodules will be discussed.