Bursting, homoclinics, period doublings and blue sky catastrophes in neural models.

W. Govaerts*

October 15, 2001

Bursting is an important behaviour pattern of nerve cells. In mathematical models it is often observed that, as a single parameter varies, the number of spikes per burst increases to a high number, followed by an irregular, apparently chaotic behaviour. Afterwards the system returns to a simpler pattern.

Numerically, spiking and bursting lead to stiff differential equations and difficult boundary value problems. Sophisticated software tools such as AUTO and CONTENT allow to detect and compute many phenomena that are common in neural models but they are very time - consuming because it is necessary to do all computations to high precision.

We study their application and the difficulties involved during the transition from bursting to chaotic behaviour to periodic spiking in two examples of the Fold/Homoclinic type of bursters. In the Morris - Lecar equations we deduce from the work of D. Terman that the system actually contains a *blue sky* catastrophe. Extensive numerical computations further strongly suggest that the model also contains a *homoclinic doubling* cascade in the natural parameters.

In the classical model of Plant for the R - 15 pacemaker neuron of *Aplysia*, the sea hare we describe a situation qualitatively similar to the Fold/Homoclinic bursting in the Morris - Lecar model. The transition from bursting to chaotic behaviour does not represent a *blue sky* catastrophe but for practical purposes is very similar. The transition from periodic spiking to chaotic behaviour is again related to the *homoclinic doubling* cascade.

*Department of Applied Mathematics and Computer Science, University of Gent, Krijgslaan 281 - S9, B - 9000 Gent