Risk-Neutral Valuation in Practice:
Implementing a Hedging Strategy for Segregated Fund Guarantees

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Hedging: Pros and Cons

- Pros:
  - Protection against catastrophic risk
  - Less volatile earnings
  - Potentially lower capital requirements, depending on OSFI approval
- Cons:
  - Lower expected income
  - Complexity and operational risks
  - Unlikely to be fully effective

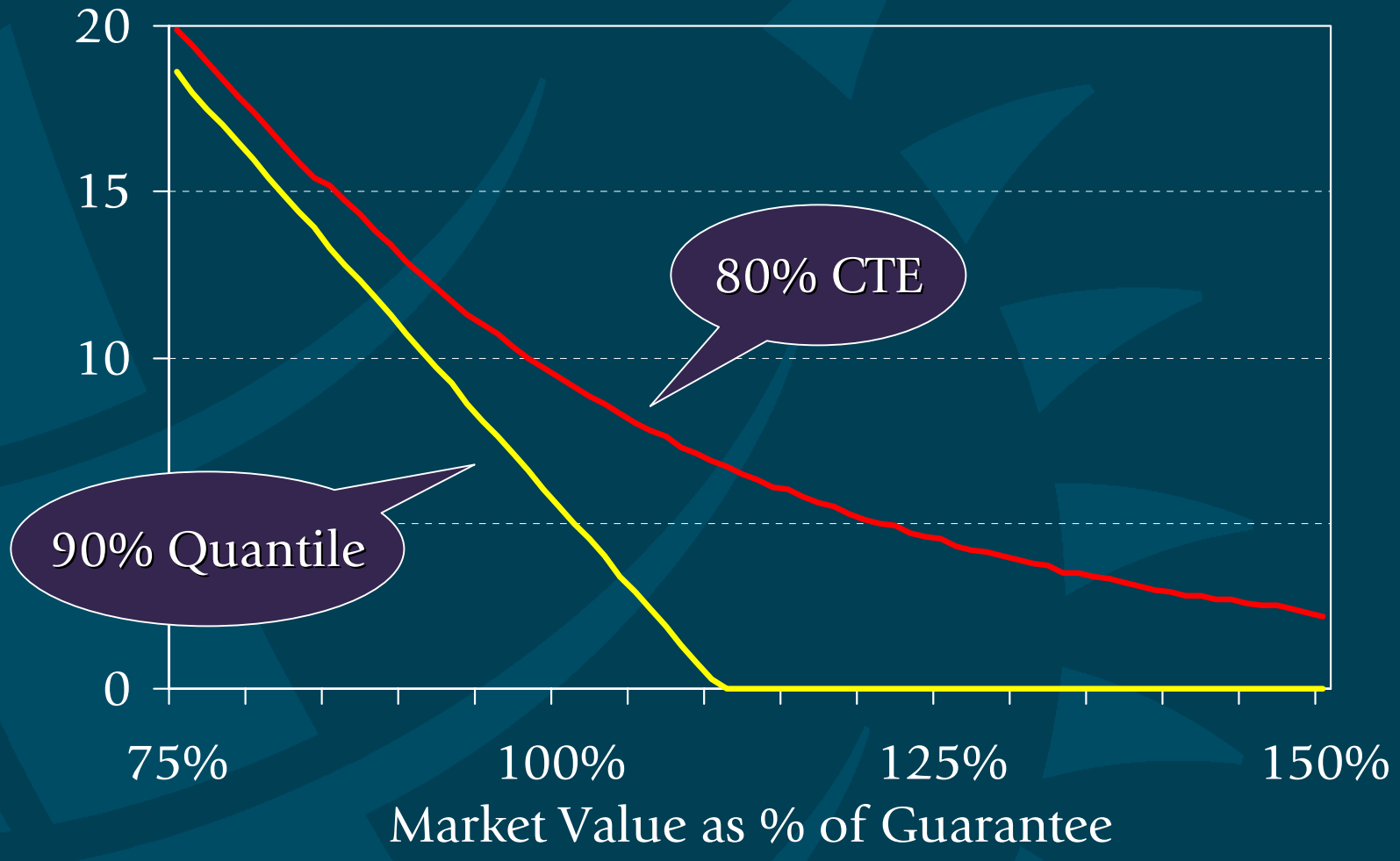
Reserving and Capital Requirements

- GAAP liabilities ("actuarial reserves")
  - Subject to guidelines established by Canadian Institute of Actuaries (CIA)
  - Explicit modelling of assets and liabilities
  - Includes implicit solvency margin
- Minimum Continuing Capital & Surplus Requirement (MCCSR)
  - Formula established by OSFI

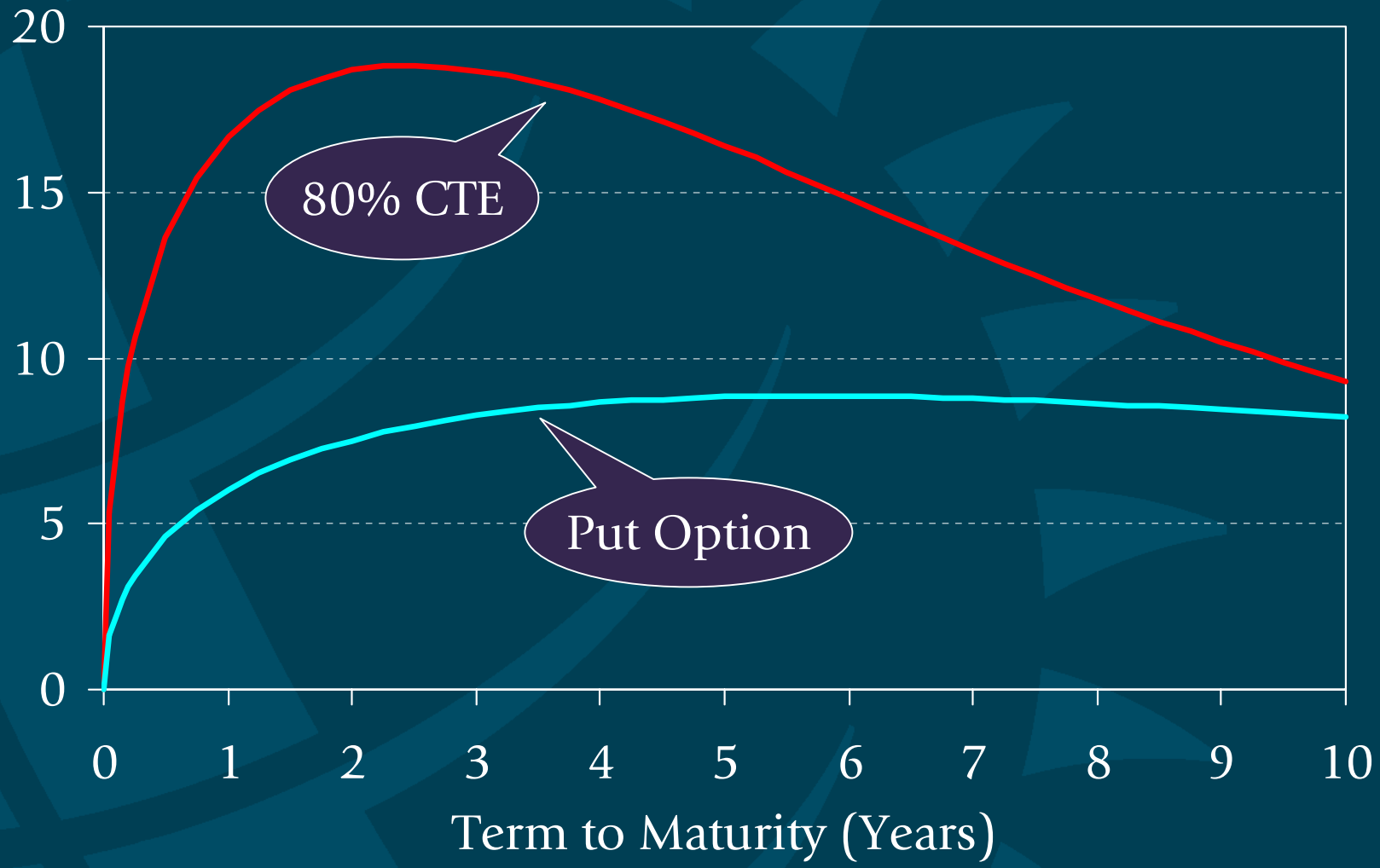
Reserving and Capital Requirements

- CIA task force report (August '00)
  - Actuarial reserves and MCCSR to be based on stochastic modelling
  - Analogous to long-dated VaR model
  - "Conditional tail expectation" (CTE x% is average of worst 100-x% of scenarios)
  - Actuarial reserves: CTE 70% to CTE 80%
  - MCCSR: Table of factors approximating CTE 95% (OSFI decision)

CTE versus Quantile Measure
Unhedged Reserves per $100 Guarantee



Unhedged Reserve versus Put Option
Per $100 Guarantee



Modelling Framework

- Hedging model
  - Based on risk-neutral valuation
  - Main purpose is to determine hedge ratios
  - Use deterministic approaches for risks that can't be hedged
- Reserving model
  - Model of real-world loss distribution, allowing for (imperfect) hedging strategy

Sun Life's Hedging Model

- Speed and simplicity are key objectives
- One-factor lognormal model
  - If guarantee applies to group of funds then aggregate balance assumed to be lognormal
- Standard trinomial tree approach
- Valuation is net of future fee income
- Mortality: deterministic function of policyholder's age

Sun Life's Hedging Model: Policyholder Behaviour

- Deterministic withdrawal rate
- Anti-selective lapses
  - Type of American option, exercised if $f_t < 0$
  - Optimal behaviour: replace f_t by 0
  - Sub-optimal behaviour: replace f_t by $f_t e^{-k\Delta t}$
- Discretionary resets
  - Equivalent to penalty-free lapse and re-entry
  - If replacement contract is fairly priced ($f_t = 0$) then can model as anti-selective lapse

Hedging Instruments

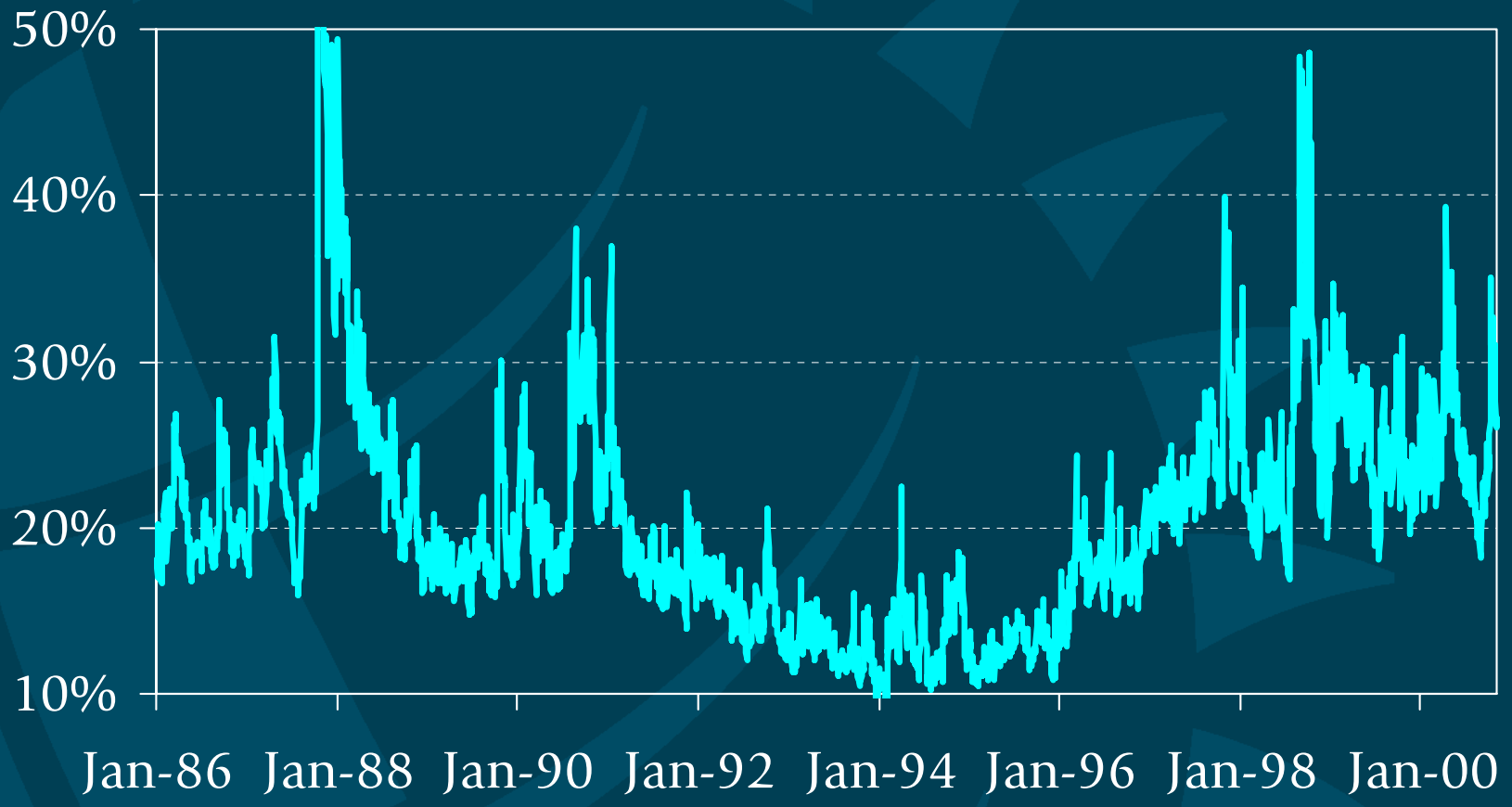
- Stock index futures and options
- Single stock options
  - Single stock futures soon to be available
- Interest rate swaps or futures
  - 50 b.p. move in 10-year interest rate has same effect as 10% stock market move
- Currency futures and options
  - CAD value of foreign funds may decline but local currency value may be unchanged

Long-Term Implied Volatility Risk

- Strategy is to use short-dated options to hedge gamma exposure
  - Protects against unexpected volatility during option term
  - Positions will have to be renewed at uncertain future implied volatility
- Model requires subjective estimate of long-term implied volatility

Implied Volatility

CBOE VIX Index, Jan 1986 to Oct 2000



Basis Risk

- Underlying funds are actively managed
  - No futures or options available
  - Fund composition generally unknown and changes over time
- Use linear regression of fund returns against basket of indices and single stocks
  - Combination of TSE60, Nortel and T-bills works well for many Canadian equity funds
  - Use exponentially-weighted moving average

Policyholder Behaviour Risk

- Model makes assumptions about withdrawals, lapses, resets, fund switches
- Behaviour is rarely 100% optimal and may change over time
- Little if any useful data
- Unexpected behaviour can't be hedged

Interest Rate Risk

- Model uses deterministic interest rate assumptions
  - Based on current forward yield curve
- Treat as a source of outside model risk
  - Duration-match using swaps or futures

Reserving Model

- Reserves = liabilities per hedging model + provision for adverse deviations
- Sources of adverse deviations:
  - Differences between target hedge ratios and actual hedge positions
  - Investment-related risks that can't be hedged (basis risk, long-term implied volatility risk)
  - Unexpected policyholder behaviour and mortality

Reserving Model

- Investment-related risks: Model stochastically and reserve at CTE 80%
  - In principle, should model hedge portfolio over entire lifetime of contract
  - Impractical for non-static hedge portfolio
  - Model static portfolio for a short holding period (1 to 3 months) and apply multiplier
- Other risks: Model deterministically based on conservative assumptions