

The future is not guaranteed

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Outline

- Landscape
- Why buy a life annuity?
- Alternatives
 - Annuity Overlay Fund
 - Group Self-Annuitization Scheme
- Comparison

Landscape

- Decline of DB schemes
- Cost of life annuities
- Annuity puzzle

Retirement choices

Drawdown

Life annuity

Retirement choices

Drawdown

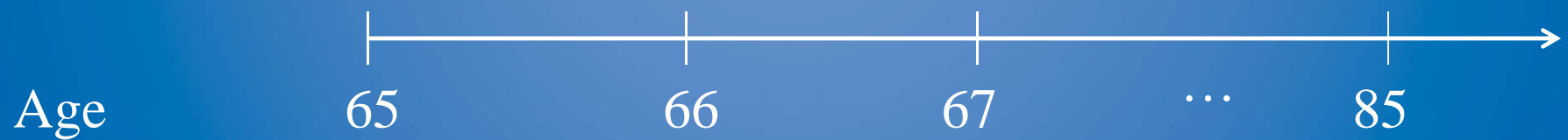
Life annuity

Alternatives

Drawdown (2% p.a.)

Assets \$100

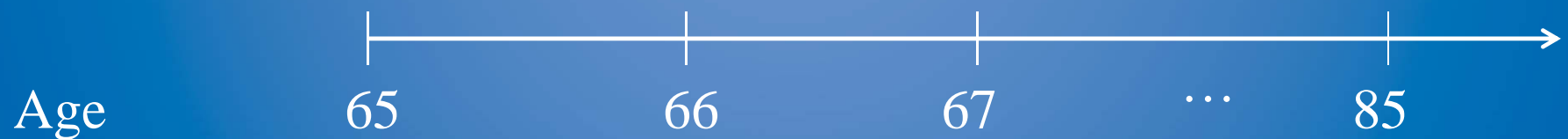
Consumption



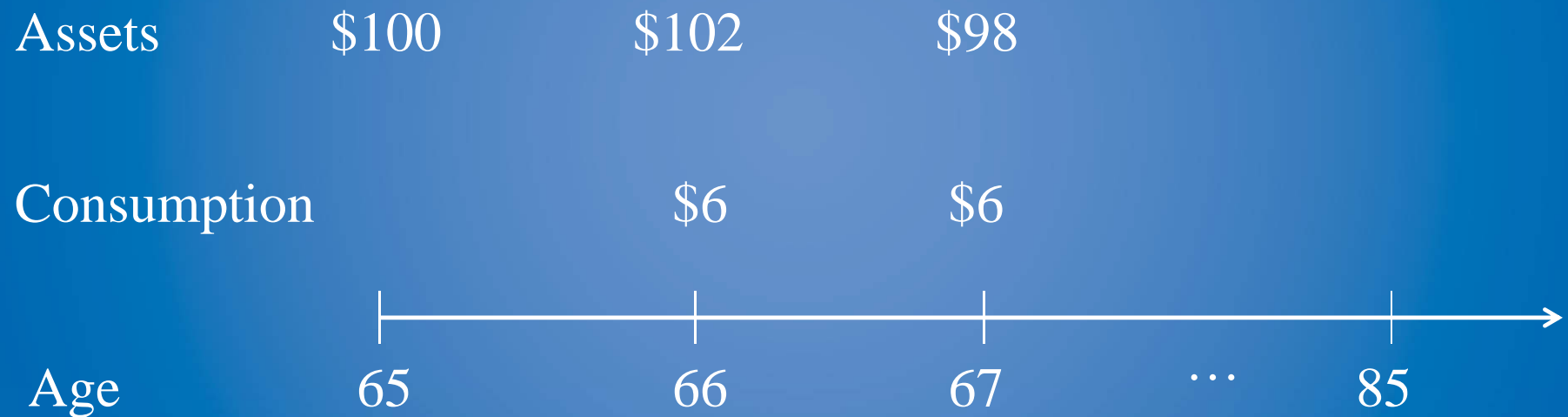
Drawdown (2% p.a.)

Assets \$100 \$102

Consumption \$6



Drawdown (2% p.a.)



Drawdown (2% p.a.)

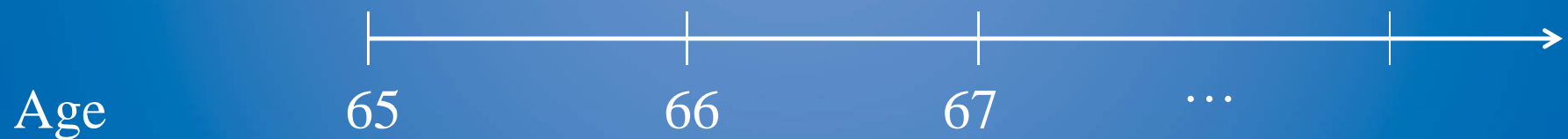


Life annuity (2% p.a.)

With 12% loading over fair value.

Pay insurer \$100

Consumption



Life annuity (2% p.a.)

With 12% loading over fair value.

Pay insurer \$100

Consumption

\$6

\$6

...

>+0

Age

65

66

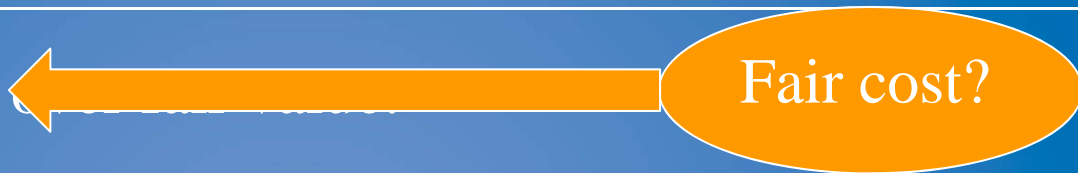
67

...

65+T

Life annuity (2% p.a.)

With 12% loading



Pay insurer \$100

Consumption

\$6

\$6

...

>+0

Age

65

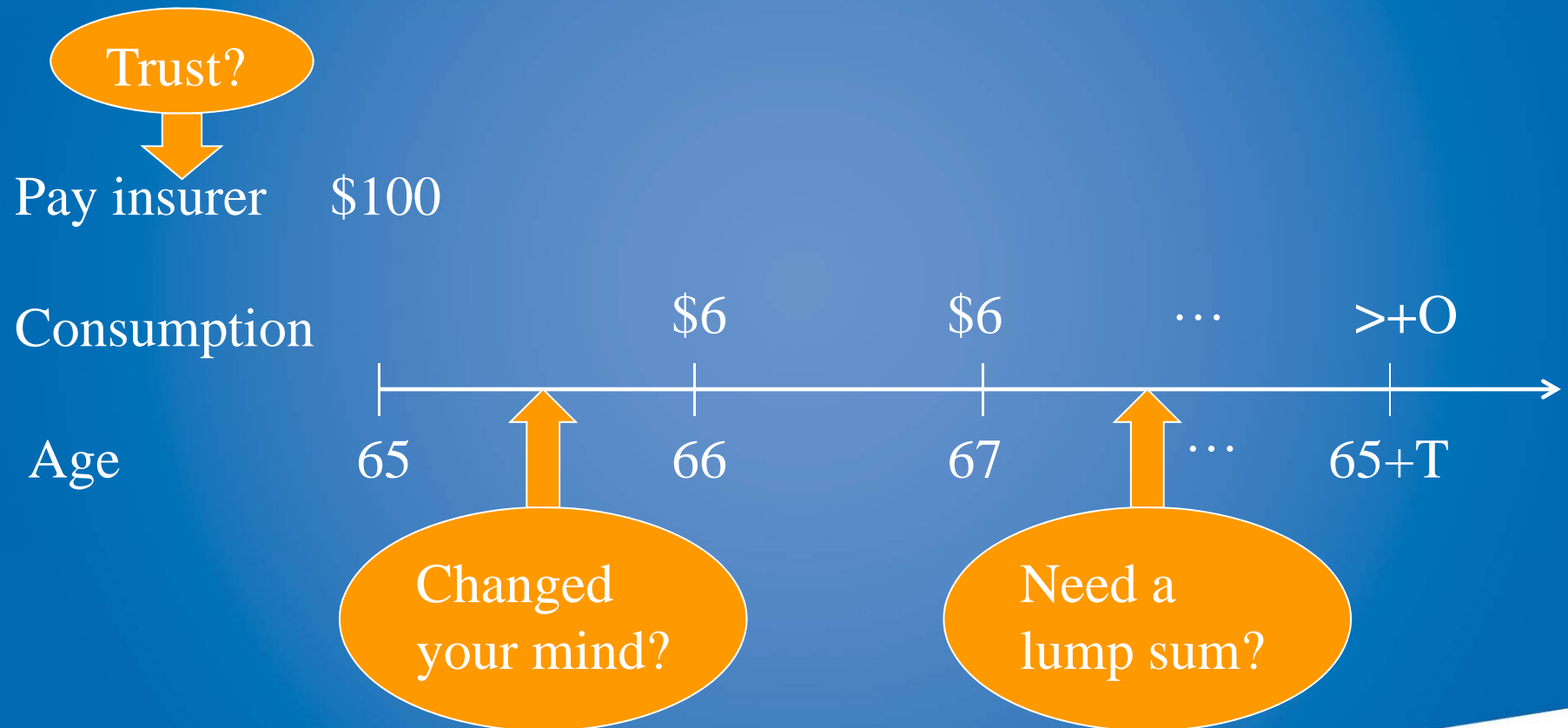
66

67

...

65+T

Life annuity

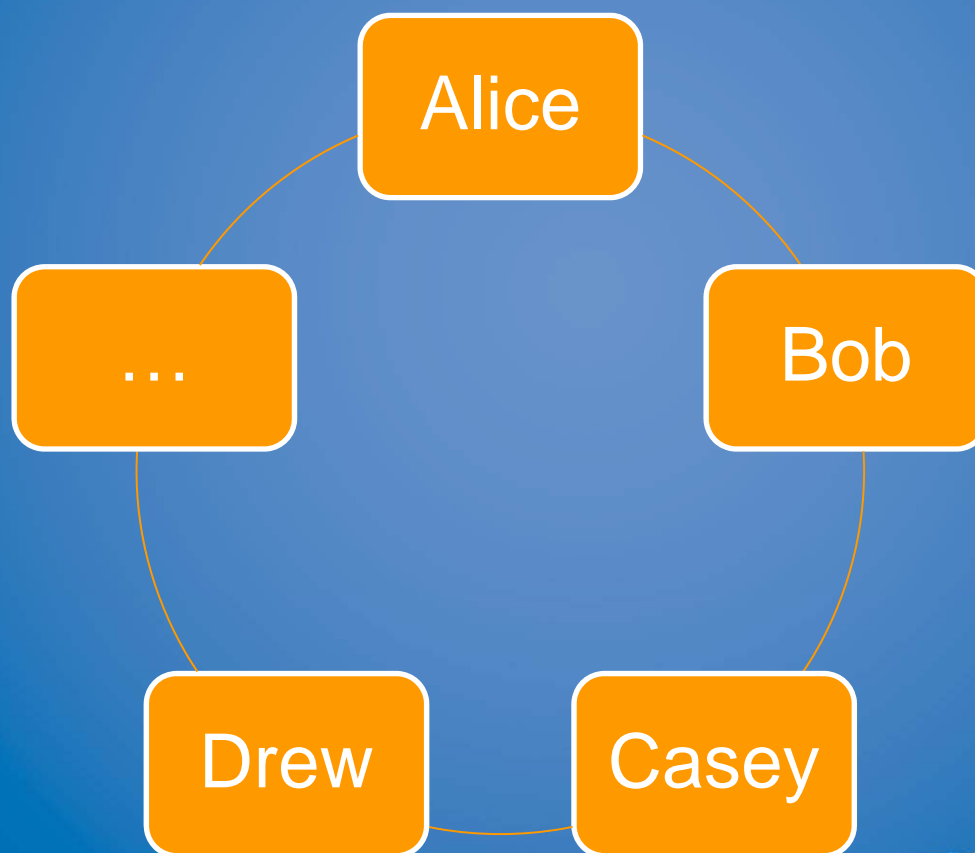


Life annuity

- Attractive for some, but not for all.
- Can people still benefit from sharing mortality risk without buying a life annuity?

Annuity overlay fund

(Donnelly, Guillén, Nielsen 2013)



Annuity overlay fund (2% p.a.)

Assets \$100

Mortality
credit

Consumption



Annuity overlay fund (2% p.a.)

Assets	\$100	\$102
Mortality credit		\$0.78
Consumption		\$6.72



Annuity overlay fund (2% p.a.)

Assets	\$100	\$102	\$97.98	...	\$34.91
Mortality credit		\$0.78	\$0.87	...	
Consumption		\$6.72	\$6.72	...	
Age	65	66	67	...	85



Choose consumption

Assets \$100 \$102 \$94.63

Mortality
credit

\$0.78 \$0.85

Consumption

\$10 **\$50**



Leave when you want



Mortality credit



Mortality credit

Proportional to:

Instantaneous probability of death

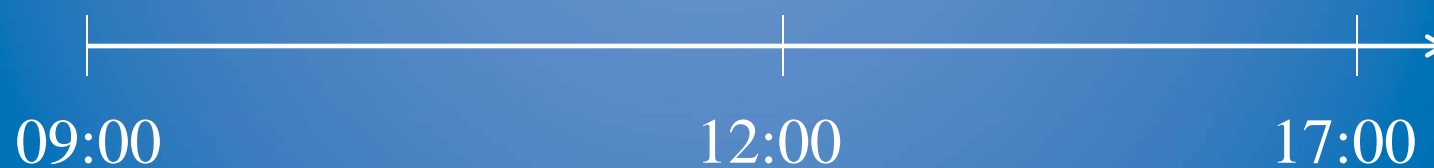
X

Fund value

Mortality credit

Alice \$100

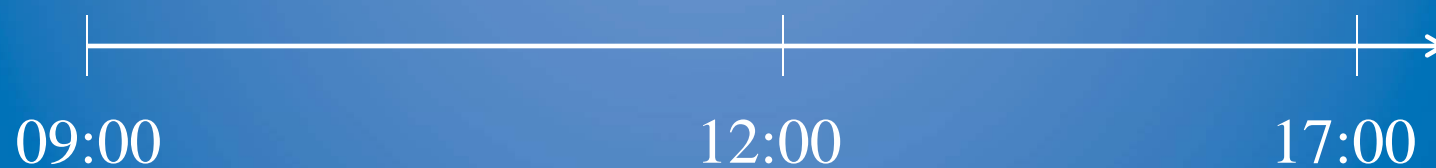
Bob \$300



Mortality credit

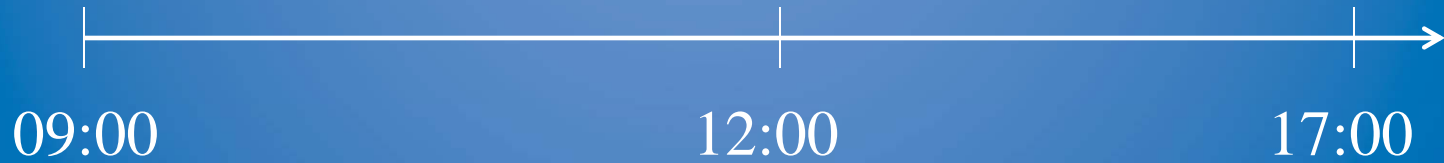
Alice \$100 $>+0$

Bob \$300 $0 \nabla \wedge$

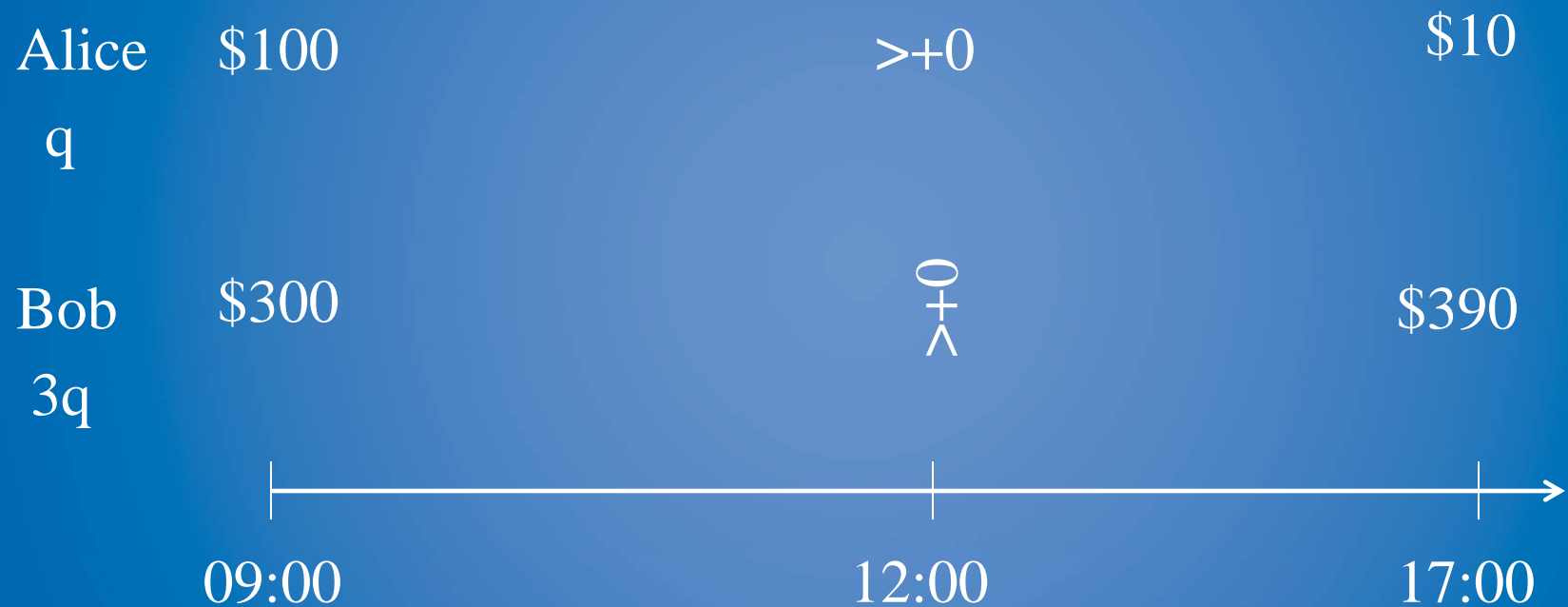


Mortality credit

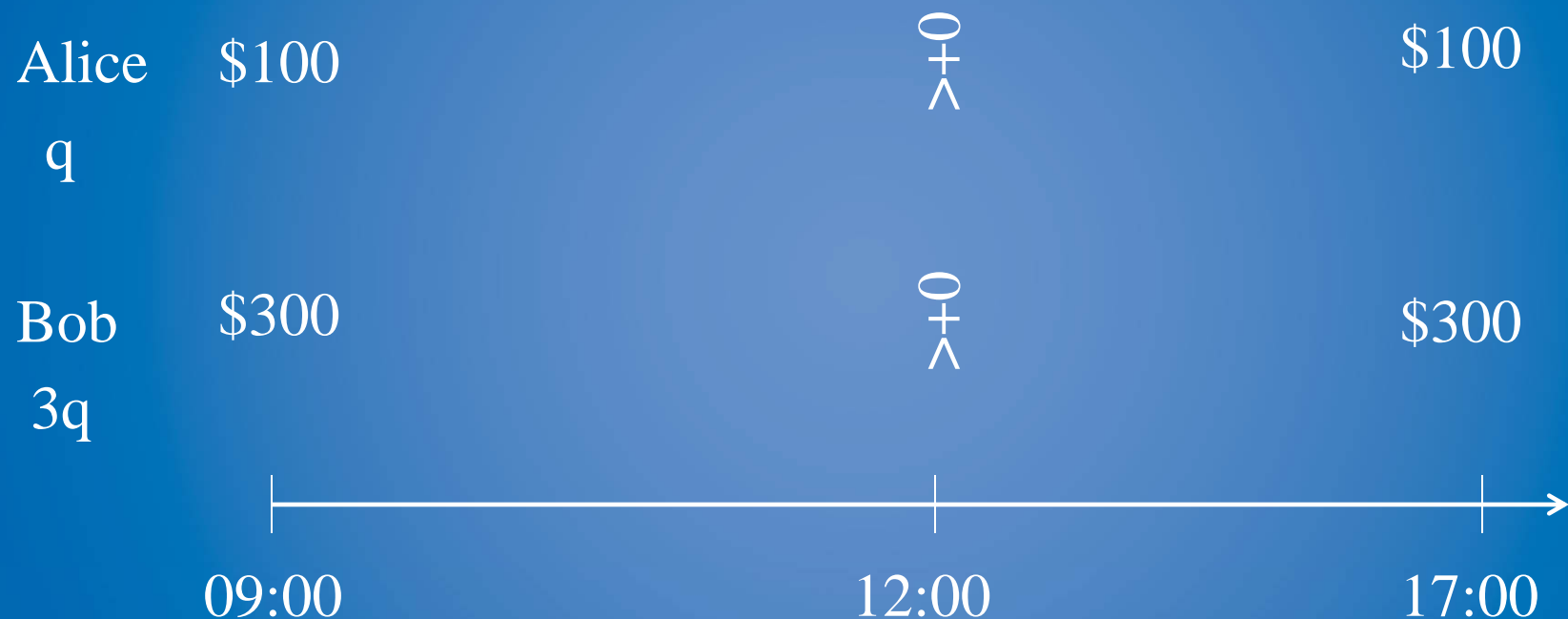
Alice	\$100	$>+0$
q		
Bob	\$300	$0 \nabla \wedge$
$3q$		



Mortality credit



Mortality credit



Mortality credit

- Amount and frequency depends on the group.
- Mortality credit always non-negative for survivors.

Annuity overlay fund - features

- Any heterogeneous group
- Contribution upon death
- Actuarially fair at all times

Annuity overlay fund - implications

- Individuals retain investment control
- Individuals decide how much to consume
- Split investment from mortality: cost transparency

Numerical experiments

- How willing are you to accept the mortality credit volatility?
- Assume Black-Scholes financial market:
 - Risk-free interest rate $r > 0$.
 - Risky asset price dynamics:

$$dS_t = \mu S_t dt + \sigma S_t dZ_t$$

Numerical experiments

$$dW_t = (r + \pi_t(\mu - r))W_{t-} dt + \sigma\pi_t dZ_t + dG_t$$

Numerical experiments

$$dW_t = (r + \pi_t(\mu - r))W_{t-} dt + \sigma\pi_t dZ_t + dG_t$$

Small
change
in
wealth

Return due to
investment in
financial market

Return due to
mortality risk
sharing

Numerical experiments

As number of members becomes infinite,

$$dW_t = (r + \pi_t(\mu - r))W_{t-} dt + \sigma\pi_t dZ_t + dG_t$$

Small
change
in
wealth

Return due to
investment in
financial market

Return due to
mortality risk
sharing

Numerical experiments

As number of members becomes infinite,

Instantaneous probability of death

$$dW_t = (r + \pi_t(\mu - r))W_{t-} dt + \sigma\pi_t dZ_t + \lambda_t W_{t-} dt$$

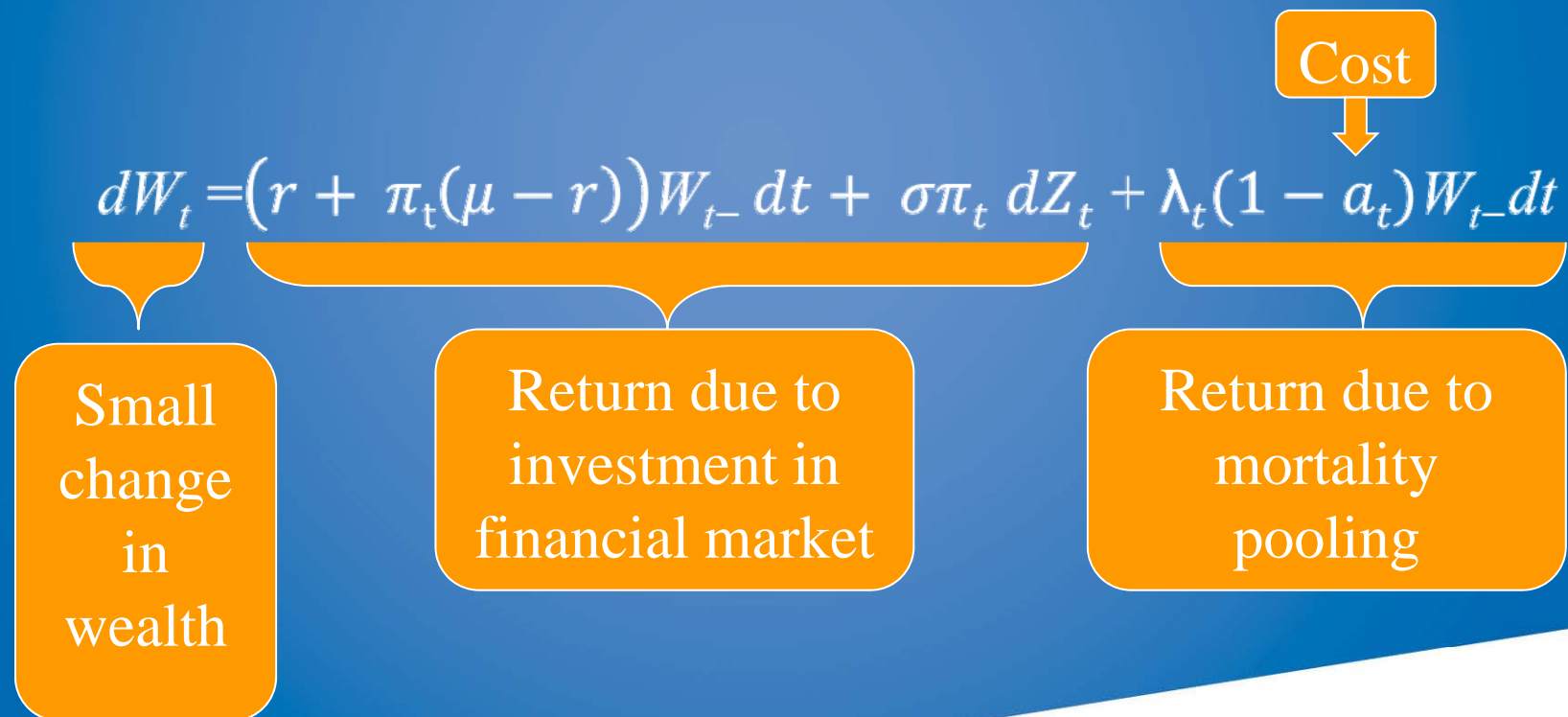
Small
change
in
wealth

Return due to
investment in
financial market

Return due to
mortality risk
sharing

Numerical experiments

Insurer's equivalent to infinite fund



Numerical experiments

Finite annuity overlay fund:

$$dW_t = (r + \pi_t(\mu - r))W_t dt + \sigma\pi_t dZ_t + dG_t$$

Insurer equivalent to infinite annuity overlay fund:

$$dW_t^\infty = (r + \pi_t^\infty(\mu - r))W_t^\infty dt + \sigma\pi_t^\infty dZ_t + \lambda_t(1 - a_t)W_t^\infty dt$$

Choose π_t^∞ so same volatility of return on wealth.

Then equate expected returns to find breakeven costs a_t^* .

Numerical experiments

- Financial market parameters: $r = 0.02$, $\mu = 0.06$, $\sigma = 0.20$.
- Gompertz mortality law: $\lambda_x = \frac{1}{9.98} e^{(x-86.85)/9.98}$
- Invest 25% of wealth in risky asset in finite annuity overlay fund.

Numerical experiments



Numerical experiments

Participants	Total number of participants in fund	Breakeven costs (% of wealth)
Old Spenders	300	<0.5% p.a.
Young Savers	300	<0.05% p.a.
Combined portfolio	300	< 0.75% p.a.

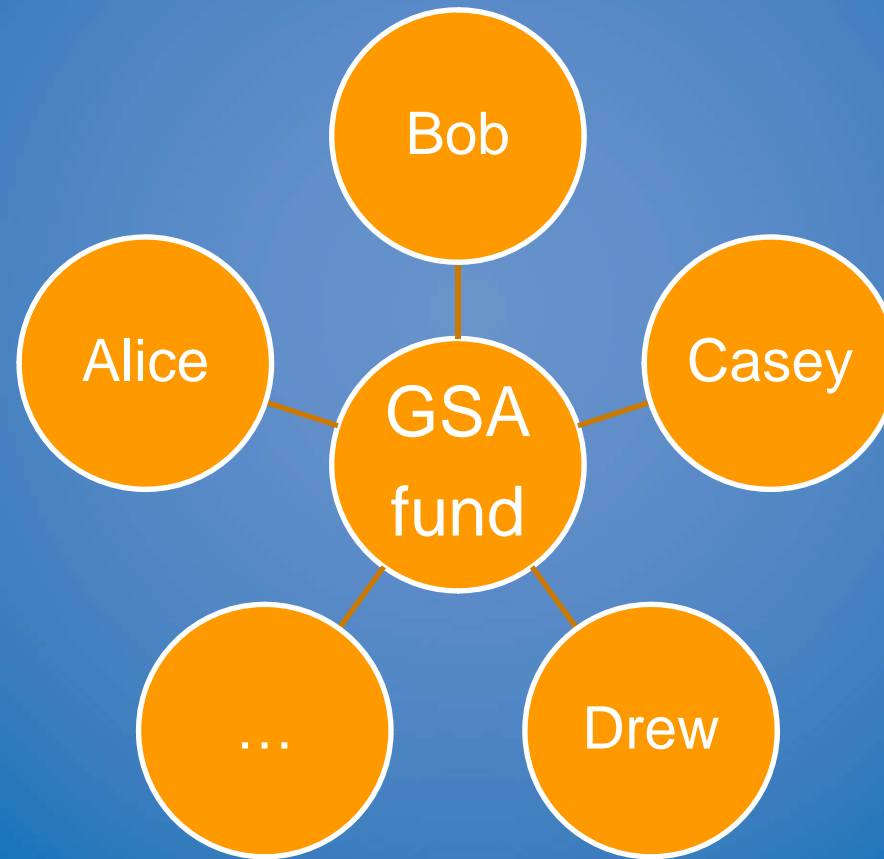
Practical questions

- Purpose of the fund?
- Conditions on fund exit and/or withdrawals?
- Conditions on investment strategies?
- Determination of mortality probabilities.
- Asset sales upon death – legal issues/time.
- Asset valuations – e.g. illiquid assets.

Group self-annuitization (GSA) scheme (Piggott, Valdez and Detzel 2005)

- Purpose: provide consumption stream to participants.
- Similar to a life annuity, without the guarantee.

GSA




GSA – participant's view

Pay scheme \$100

Consumption \$6.72 \$6.72 ... >+0

Age 65 66 67 ... 65+T




GSA– scheme perspective

Assets \$10,000 \$10,200 \$9,723 ... \$1,618

Payments out \$667 \$662 ... \$281

Age 65 66 67 ... 85



GSA

- Share mortality risk.
- Same investment strategy for all participants.

GSA calculation

For each participant,

This year's consumption payment

=

Last year's consumption payment

X

Mortality adjustment

X

Investment return adjustment

GSA calculation

For each participant,

This year's consumption payment

=

Last year's consumption payment

X

Mortality adjustment

X

Investment return adjustment

Same
adjustment
for all
participants

GSA calculation

- Adjustments compare actual experience over the year to expected experience over the year

GSA - features

- Any heterogeneous group
- Contribution upfront: pays consumption stream
- Not actuarially fair but may be only significant for highly heterogeneous groups (Sabin 2010, Donnelly 2013).

GSA - implications

- Assets centrally managed
- Consumption calculation pre-determined
- Cost transparency

Quick comparison

- Life annuity contract
- Annuity overlay fund
- Group self-annuitization (GSA) scheme

Quick comparison

	Life annuity	GSA	Annuity overlay
Who bears mortality risk?	Insurer	Group	Group
Mortality pooling?	✓ (Indirect)	✓ (Direct)	✓ (Direct)
Mortality guarantee?	✓	x	x

Quick comparison

	Life annuity	GSA	Annuity overlay
Who bears investment risk?	Insurer	Individual	Individual
Investment guarantee?	✓	x	x
Premium/ contribution paid	Upfront	Upfront	Upon death

Quick comparison

	Life annuity	GSA	Annuity overlay
Consumption stream?	✓	✓	✓ (individual's choice)
Lump sum withdrawals?	x	x	✓
Exit before death?	x	x	✓

Quick comparison

	Life annuity	GSA	Annuity overlay
Costs transparent?	x	✓	✓
Individual investment control	x	x	✓
Actuarially fair?	?	x	✓

Conclusion

- Practical implementation.
- Further questions: can we share investment risk across time?
- Challenge: construct robust, transparent, easy-to-understand pension schemes.

References

- Bringing cost transparency to the life annuity market.
By C. Donnelly, M. Guillén and J.P. Nielsen (2013). *Working paper*.
- Actuarial fairness and solidarity in pooled annuity funds.
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- The simple analytics of a pooled annuity fund.
By J. Piggott, E.A. Valdez and B. Detzel (2005). *Journal of Risk and Insurance*, 72(3) 497–520.
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Thank you!

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