Sharing Longevity Risk in Life Annuity Contracts

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Annual IFID Centre Conference 2013

Toronto, Fields Institute, November 28

Agenda

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2. Options for the payout-phase
3. Managing longevity risk
4. Options embedded in Group Self-Annuitization (GSA) strategies
5. Longevity/Mortality-linked annuities
6. Pricing the embedded options
7. Final remarks & further research

Motivation

- **Long-term demographic (ageing) trends will drive to important changes in the income mix of retirees, due to**
  - § **Reforms in public social security systems**
    - o **Increase in contribution rates, reduction in pension/salary ratios or accrual rates, increase in the retirement age,...**
    - o **PAYGO ð NDC Pension systems**
  - § **Market trend away from DB to DC pension schemes**
  - § **Mobility of the workforce, labour market uncertainty,...**
  - § **Increasing pressure on public finances**
- **The reduction in the retirement income that is “guaranteed” means that individuals will have to become more self-reliant**

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Options for the payout-phase

- **Main options for the payout phase**

  1. **Self-annuitization (lump-sum payments)**
  2. **Programmed withdrawals**
  3. **Group Self-Annuitization (GSA)**
  4. **Longevity/Mortality linked annuities**
  5. **Traditional life annuities**

- **The sharing of biometric (longevity/mortality) and market risks (interest rate, inflation, equity, credit, liquidity,…) between individuals and annuity providers is different between agreements**

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Traditional life annuities

- **Under traditional whole life annuities the annuitant is entitled to receive a specified annuity benefit as long as he/she lives,**

  1. **independently of his/her lifetime (*individual longevity risk*)**
  2. **whatever the lifetimes of the annuitants in the annuity portfolio or pension fund (*aggregate/systematic longevity risk*)**
  3. **Whatever the investment yield obtained by the annuity provider (*rate of return guarantee*)**

- **Other options could be added to this product (capital protection)**
- **Despite their appealing characteristics and economic foundations, the significance of annuity markets is still limited**

Annuity markets constraints: Demand

- The existence of substitutes, namely the «crowding-out» effect induced by "generous" public pensions
- "If I die soon after I retire, the annuity provider will keep my fund"
- Bequest motive, iliquidity
- Annuities are perceived as expensive
- The underestimation of the personal longevity, myopia
- Seen as a financial investment in the accumulation phase
- Insufficient tax incentives
- Lack of understanding of the properties of annuities
- Lack of trust in financial institutions (emerging economies)

Annuity markets constraints: Supply

- **Adverse selection problems ð safety loadings**
- **Lack of high-quality information on prospective mortality tables (*their pricing requires stochastic mortality models*)**
- **Inexistence or insufficiency of assets with which to back the long-term promises and risks (interest rate, longevity, liquidity risks) represented by annuities**
- **Regulatory capital requirements (Solvency II)**

Managing longevity risk

- **Annuity providers (pension funds) have roughly two approaches in managing longevity risk**

  1. **Transfer the risk to another counterparty**
     - **Insurance-based solutions**
     - **Capital market solutions**
     - **Product design**

  2. **Hedge the risk while retaining it**
     - **Portfolio diversification**
     - **Pricing policy**
     - **Product design**

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Managing longevity risk

Insurance-based solutions	Capital markets-based solutions
§ Annuities § Buy-in § Buy-out § Reinsurance arrangements	§ Longevity Bonds § Extreme mortality structures (Mortality Bonds) § *q*-Forwards § Longevity Swaps § Longevity Options § Life insurance securitization § Collateralised longevity obligations

Product design & Pricing policy

- **Conservative pricing policy (*higher safety loadings*)**
- **Price differently among individuals according to their specific risk factors (e.g., smoking) ð *contingency loadings***
- **Reduce the level of investment profit participation**
- **Annuities with premiums indexed to the evolution of longevity**
- **Participating (*with-profit* GAR) product that shares part of the emerging profit/loss among surviving policyholders**
- **Annuities with benefits linked to mortality/longevity**

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Group Self-Annuitization (GSA)

- **Piggott *et al*. 2005, Sherris and Qiao 2011, Valdez *et al*. 2006, Wadsworth *et al* 2001, van de Ven and Weale 2008, Lüty *et al* 2001**
- **Under a GSA pool participants, are insured against the idiosyncratic risk but bear all of the systematic longevity risk**
- **Similarities with standard annuities**
  - **Mortality credits are redistributed among the survivors**
  - **participants renounce to bequest or liquidity motives**
  - **the decision to purchase pooled annuity fund units is irreversible**
  - **the advantages of investment diversification are profited**
- ***Crucial difference*: benefit payments are linked to the mortality experience of the group according to an adjustment coefficient**

Group Self-Annuitization (GSA)

1. The pool starts at time $t=0$ with an initial size of l_x homogeneous retirees in the sense of identical age, gender and cohort, identical monetary amounts and identical risk exposures
2. The annuity arrangement is offered to individuals in exchange for a single upfront premium
3. The annuity provides an initial level benefit B_0, paid once a year, calculated using an annuity factor accounting for expected mortality improvements and a flat interest rate
4. At any future moment t, the benefit payment will be determined by

$$B_t = B_0 \times \underbrace{\left(\frac{{}_t p_x^{F_0}}{{}_t \tilde{p}_x} \right)}_{MCA} \times \underbrace{\frac{\prod_{i=1}^{t} \left(1+\tilde{r}_i\right)}{\left(1+r_0\right)^t}}_{IRA}$$

GSA: embedded options

- **Assume, without loss of generality, that we disregard the IRA component in the GSA adjustment formula**

- **Benefit payments at time *t* can be expressed as:**

1. $$f_t(\tilde{B}_t) = B_0, \qquad \text{if} \quad \frac{{}_t p_x^{F_0}}{{}_t \tilde{p}_x} = 1$$

2. $$f_t(\tilde{B}_t) = \min(B_0, \tilde{B}_t) = B_0 - \underbrace{\max\left(B_0 - \tilde{B}_t ; 0\right)}_{\text{European put option}}$$

$$= B_0 - \max\left(B_0 \left(1 - \frac{{}_t p_x^{F_0}}{{}_t \tilde{p}_x} \right) ; 0 \right)$$

$$= B_0 \left(1 - \max\left(1 - \frac{{}_t p_x^{F_0}}{{}_t \tilde{p}_x} ; 0 \right) \right), \qquad \text{if} \quad \frac{{}_t p_x^{F_0}}{{}_t \tilde{p}_x} < 1$$

GSA: embedded options

- **Benefit payments at time *t* can be expressed as (cont'd):**

3. $$f_t(\tilde{B}_t) = \max(B_0, \tilde{B}_t) = B_0 + \underbrace{\max\left(\tilde{B}_t - B_0;0\right)}_{\text{European call option}}$$

$$= B_0\left(1+\max\left(\frac{{}_tp_x^{F_0}}{{}_t\tilde{p}_x} - 1;0\right)\right), \quad \text{if } \frac{{}_tp_x^{F_0}}{{}_t\tilde{p}_x} > 1$$

- **Compared with traditional annuities, GSA include embedded options that may reduce (increase) benefit payments in the future if the actual survivorship rates are higher (lower) than expected and thus should cost less (more) an amount equal to the option premium**

GSA: embedded options

- **The loss on the underlying standard annuity portfolio at time *t* due to longevity risk is defined as**

$$L_t = \sum_{i=1}^{l_x} B_0 \left(I_i(t) - E[I_i(t)] \right)^+ = l_x B_0 \left({}_t\tilde{p}_x - {}_t p_x^{F_0} \right)^+$$

- **The portfolio loss at time *t* redistributed among the survivors is**

$$\frac{L_t}{\tilde{l}_{x+t}} = \frac{l_x B_0}{\tilde{l}_{x+t}} \left({}_t\tilde{p}_x - {}_t p_x^{F_0} \right)^+ = B_0 \times \max\left(\left(1 - \frac{{}_t p_x^{F_0}}{{}_t\tilde{p}_x} \right); 0 \right)$$

- **The loss “inherited” by each surviving policyholder includes a put option that depends on ratio of survivorship rates**

GSA: disutility sources

- In a scenario of longevity risk, a pure GSA is expected to pay decreasing annuity benefits
- The annuitant does not know in advance the rate of return of the pool, hence it carries some upside/downside risk
- The introduction of uncertainty in annuity benefits will have implications in consumption/saving behaviour according to the individual's degree of risk aversion (e.g, CRRA ð precautionary saving ð annuity with increasing payments)
- The dispersion of future mortality (and investment) rates is ignored
- Annuity providers do not bear any kind of risk
- In a limiting situation there will be no payments for those who survive beyond the highest attainable age assumed in the life table

Longevity/Mortality-linked annuities

- **Questions that need to be addressed**
  1. **who should ultimately bear the longevity risk?**
  2. **At what price?**
  3. **How should we define the benefit adjustment coefficient?**
     - **To the mortality experienced by a specific pool of annuitants, by the general population or by a mixture of populations?**
     - **To which longevity measure (life expectancy, actuarial value of an annuity, portfolio reserves and assets)?**
     - **Retrospectively or prospectively?**
     - **With which periodicity?**

Adjustment based on the number of survivors

- **1.** **Considering the life table set at time 0, a participation rate combined with a mandatory conversion into a level life annuity at some advanced age** x_{max}

$$B_t = B_{t-1}\left(1 - a_t \times \max\left(1 - \frac{{}_t p_x^{F_0}}{{}_t \hat{p}_x};0\right)\right), \quad a_t \in [0,1], \;\; x < x_{\max}$$

  - **Annuitants bear all/part longevity risk up to x_{max}, then the annuity provider steps up; annuity provider takes the life table risk; Asymmetric contract, annuitants give up some potential upside**

- **2.** **Considering the life table set at time t**

$$B_t = B_{t-1}\left(1 - a_t \times \max\left(1 - \frac{{}_t p_x^{F_t}}{{}_t \hat{p}_x};0\right)\right)$$

Adjustment based on the number of survivors

- **3. Considering the limits set by, e.g., the 9X% confidence interval for the survival probability**

$$B_t = B_{t-1}\left(1 - a_t \max\left(1 - \frac{{}_t p_x^{UB,F_0}}{{}_t \hat{p}_x};0\right)\right)$$

  - **Only systematic risk is shared**

- **4. Or its symmetric version (Cap & Floor) (see also Denuit *et al.*, 2011)**

$$B_t = \begin{cases} B_{t-1}\left(1 - a_t \max\left(1 - \frac{{}_t p_x^{UB,F_0}}{{}_t \hat{p}_x};0\right)\right), & a_t \in [0,1],\ \frac{{}_t p_x^{F_0}}{{}_t \hat{p}_x} < 1 \\ B_{t-1}\left(1 + h_t \max\left(\frac{{}_t p_x^{LB,F_0}}{{}_t \hat{p}_x} - 1;0\right)\right), & h_t \in [0,1],\ \frac{{}_t p_x^{F_0}}{{}_t \hat{p}_x} > 1 \end{cases}$$

Adjustment based on the reference population

- **5. Variants [1] to [4] can be reformulated and be based on the relation between the expected and observed number of survivors (or survivorship rates) in the reference population (or a mixture of populations), e.g.,**

$$B_t = \begin{cases} B_{t-1}\left(1 - a_t \max\left(1 - \dfrac{{}_t p_x^{UB,F_0,POP}}{{}_t \hat{p}_x^{POP}};0\right)\right), & a_t \in [0,1],\ \dfrac{{}_t p_x^{UB,F_0,POP}}{{}_t \hat{p}_x^{POP}} < 1 \\ B_{t-1}\left(1 + h_t \max\left(\dfrac{{}_t p_x^{LB,F_0,POP}}{{}_t \hat{p}_x} - 1;0\right)\right), & h_t \in [0,1],\ \dfrac{{}_t p_x^{UB,F_0,POP}}{{}_t \hat{p}_x^{POP}} > 1 \end{cases}$$

In this case, the basis risk (adverse selection effect) is borne by the annuity provider and less idiosyncratic risk is transferred to individuals

Adjustment based on different life tables

- **6. Considering the life table set at time 0 and time *t* for the annuitants population**

$$B_t = B_{t-1}\left(1 - a_t \times \max\left(1 - \frac{{}_t p_x^{F_0}}{{}_t p_x^{F_t}};0\right)\right)$$

- **7. Or the equivalent design considering population life tables**

$$B_t = B_{t-1}\left(1 - a_t \times \max\left(1 - \frac{{}_t p_x^{F_0,POP}}{{}_t p_x^{F_t,POP}};0\right)\right)$$

Adjustment based on a_x ratios

- **8. The benefit is adjusted according to the update in the actuarial value of a life annuity, as calculated according to an annuity market life table or a population life table**

$$B_t = B_{t-1}\left(1 - a_t \times \max\left(1 - \frac{\mathrm{E}\left[a_{K_{x+t}} \mid F_0\right]}{\mathrm{E}\left[a_{K_{x+t}} \mid F_t\right]};0\right)\right)$$

Variant: ratio between remaining life expectancies

- **9. Variant, linking annuity market and population life tables**

$$B_t = B_{t-1}\left(1 - a_t \times \max\left(1 - \frac{\mathrm{E}\left[a_{K_{x+t}} \mid F_0\right]}{\mathrm{E}\left[a_{K_{x+t}} \mid F_t^{POP}\right]};0\right)\right)$$

Adjustment based on asset/reserve ratio

- **10. Adjust, symmetrically, the benefits according to the ratio between observed assets and portfolio reserves**

$$B_t = \begin{cases} B_{t-1}\left(1 - a_t \times \max\left(1 - \dfrac{A_t}{V_t^{[P]}};0\right)\right) \\ B_{t-1}\left(1 + h_t \max\left(\dfrac{A_t}{V_t^{[P]}} - 1;0\right)\right) \end{cases}$$

In this case, all of the biometric and financial risks may be borne by the annuitants, depending on the participation rate coefficient

- **11. Previous variants considering also financial market risks**

Pricing the embedded options

- A key condition for the development of longevity-linked products and markets is the development of generally agreed stochastic mortality models
  - Discrete-time (e.g., Lee-Carter model & extensions)
  - Continuous-time (SDE for the spot/forward mortality surface)

- How to determine the market price for longevity risk?
  - Risk-neutral valuation approach
  - Distortion approaches (Wang transform)
  - Use classic premium principles (e.g., standard deviation principle)
  - Sharpe ratio
  - Consumption CAPM, Mean-variance and Risk minimization strategies

Pricing the embedded options

- **European call option**

$$c_t = E^Q\left[e^{-r(T-t)}\max\left({}_t\not p_x -{}_t\, p_x^{F_0};0\right)\right]$$

- **European put option**

$$c_t = E^Q\left[e^{-r(T-t)}\max\left({}_t\, p_x^{F_0} -{}_t\not p_x;0\right)\right]$$

- **American call option**

$$C_t = \sup_{t\in z} E^Q\left[e^{-r(t-t)}\max\left({}_t\not p_{x,t} -{}_t\, p_{x,t}^{F_0};0\right)\right]$$

- **American put option**

$$P_t = \sup_{t\in z} E^Q\left[e^{-r(t-t)}\max\left({}_t\, p_{x,t}^{F_0} -{}_t\not p_{x,t};0\right)\right]$$

Stochastic mortality modelling

Model	Formula
M1 (Poisson-Lee-Carter)	$\ln(m_{x,t}) = b_x^{(1)} + b_x^{(2)} \cdot k_t^{(2)}$
M2 (Renshaw-Haberman Cohort-Lee-Carter)	$\ln(m_{x,t}) = b_x^{(1)} + b_x^{(2)} \cdot k_t^{(2)} + b_x^{(3)} \cdot g_{t-x}^{(3)}$
M3 (APC, Currie 2006)	$\ln(m_{x,t}) = b_x^{(1)} + \frac{1}{n_x} \cdot k_t^{(2)} + \frac{1}{n_x} \cdot g_{t-x}^{(3)}$
M5 (CBD)	$\operatorname{logit} q_{x,t} = k_t^{(1)} + k_t^{(2)}(x - \bar{x})$
M6 (CBD cohort)	$\operatorname{logit} q_{x,t} = k_t^{(1)} + k_t^{(2)}(x - \bar{x}) + g_{t-x}^{(3)}$
M7 (CBD quadratic age effect + cohort)	$\operatorname{logit} q_{x,t} = k_t^{(1)} + k_t^{(2)}(x - \bar{x}) + k_t^{(3)}((x - \bar{x})^2 - \hat{s}_x^2) + g_{t-x}^{(4)}$
M8 (CBD decreasing cohort effect)	$\operatorname{logit} q_{x,t} = k_t^{(1)} + k_t^{(2)}(x - \bar{x}) + g_{t-x}^{(3)}(x_c - x)$

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Relational models for mortality

- **If the use of internal models for longevity risk is recommended (or allowed), you may need to resort to Brass-type relational models for life table construction**
- **General formulation**

$$g\left(\hat{q}_{x,t}\right) = b_0 + f_1\left(g\left(\hat{q}^{REF}_{x,t}\right)\right) + f_2\left(x\right) + e_{x,t}$$

***g*(.) = log, logit, log(log),...**

- **Parameters estimated by WOLS or ML**
- **If you use Monte Carlo Simulation (MCS) methods to price options you may need a *double-bootstrap approach***

Stochastic mortality modelling

- **Model the survival probability using affine-jump diffusion processes**

$$dX_t = d(t, X_t)dt + s(t, X_t)dW_t + dJ_t\,, \quad J_t = \sum_{i=1}^{N_t} e_i, \quad e_i \text{ i.i.d}$$

- ***Example*: Feller equation**

$$dm_{x+t}(t) = a m_{x+t}(t)dt + s\sqrt{m_{x+t}}\,dW(t) + dJ_t$$

- ***dJ_t* is a Poisson process with constant jump-arrival intensity; jump sizes follow a double asymmetric exponential distribution**

$$f(z) = p_1\left(\frac{1}{u_1}\right)e^{-\frac{1}{u_1}} I_{\{z\geq 0\}} + p_2\left(\frac{1}{u_2}\right)e^{\frac{1}{u_2}} I_{\{z<0\}}$$
$$p_1, p_2, u_1, u_2 \geq 0, \quad p_1 + p_2 = 1$$

Stochastic mortality modelling

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$$dX_t = d(t, X_t)dt + s\,(t, X_t)dW_t + dJ_t\,, \quad J_t = \sum_{i=1}^{N_t} e_i, \quad e_i \text{ i.i.d}$$

- ***Example*: Feller equation**

$$dm_{x+t}(t) = am_{x+t}(t)dt + s\sqrt{m_{x+t}}\,dW(t) + dJ_t$$

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$$p_1, p_2, u_1, u_2 \geq 0, \quad p_1 + p_2 = 1$$

Stochastic mortality modelling

- **Assuming that the survival probability can be represented by an exponential affine function, we can get a closed-formula solution**

$$ {}_{T-t}p_{x+t}(t) = \exp\left(A(t) + B(t) \cdot m_{x+t}(t) \right) $$

with

$$ B(t) = \frac{1-e^{kt}}{a_0 + a_1 e^{kt}}, \qquad k = \sqrt{a^2 + 2s^2}, \quad a_0 = \frac{a+k}{2}, \quad a_1 = \frac{k-a}{2}, $$

$$ A(t) = hp_1 \left\{ \frac{a_0 t}{(a_0 - n_1)} + \frac{n_1(a_0+a_1)[\ln(a_0+a_1) - \ln(a_0 - n_1 + (a_1+n_1)e^{kt}}{k(a_0-n_1)(a_1+n_1)} \right\} $$

$$ + hp_2 \left\{ \frac{a_0 t}{(a_0+n_2)} + \frac{n_2(a_0+a_1)}{k(a_1-n_2)(a_0+n_2)} \left[-\ln(a_0+a_1) \right. \right. $$

$$ \left. \left. + \ln(a_0 + n_2 + (a_1 - n_2)e^{kt} \right] \right\} - ht $$

Final remarks

- Providers of traditional life annuities are exposed to idiosyncratic and systematic longevity risks and financial market risks
- Designing longevity/mortality-linked annuities with risk sharing mechanisms is one of the potential solutions to manage the risk
- When designing the contract, we should take into account the nature of the risk (idiosyncratic/systematic), the capacity of annuitants to absorb it and the trade-off between risk and the price of the embedded options
- Pricing the contract requires sound stochastic mortality models
- Pricing the embedded options requires estimating the market price of longevity risk and may demand the use of MCSM

Further research

- Determine the "optimal" level of annuitization and risk sharing in longevity-linked annuities
- Assess the price and the risks of the alternative contract specifications using Monte Carlo Simulation methods
- Estimate the impact of these contract designs on capital requirements
- Conduct an enquiry to perceive the receptivity of individuals to these risk sharing mechanisms

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THANK YOU

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Toronto, IFID Centre, Fields Institute, November 28, 2013