Opportunity Cost Techniques and Fulfillment Tie-Breaking

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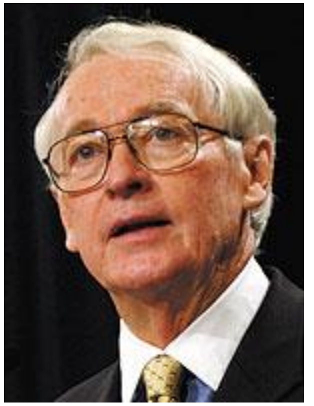
Motivation

- James Q. Wilson, Ph.D.
- Broken Windows Theory
  - Focus on the small, reap benefits on the big.
  - Randomized experiment[1] in Lowell, MA resulted in a statistically significant 20% drop in police calls for service.

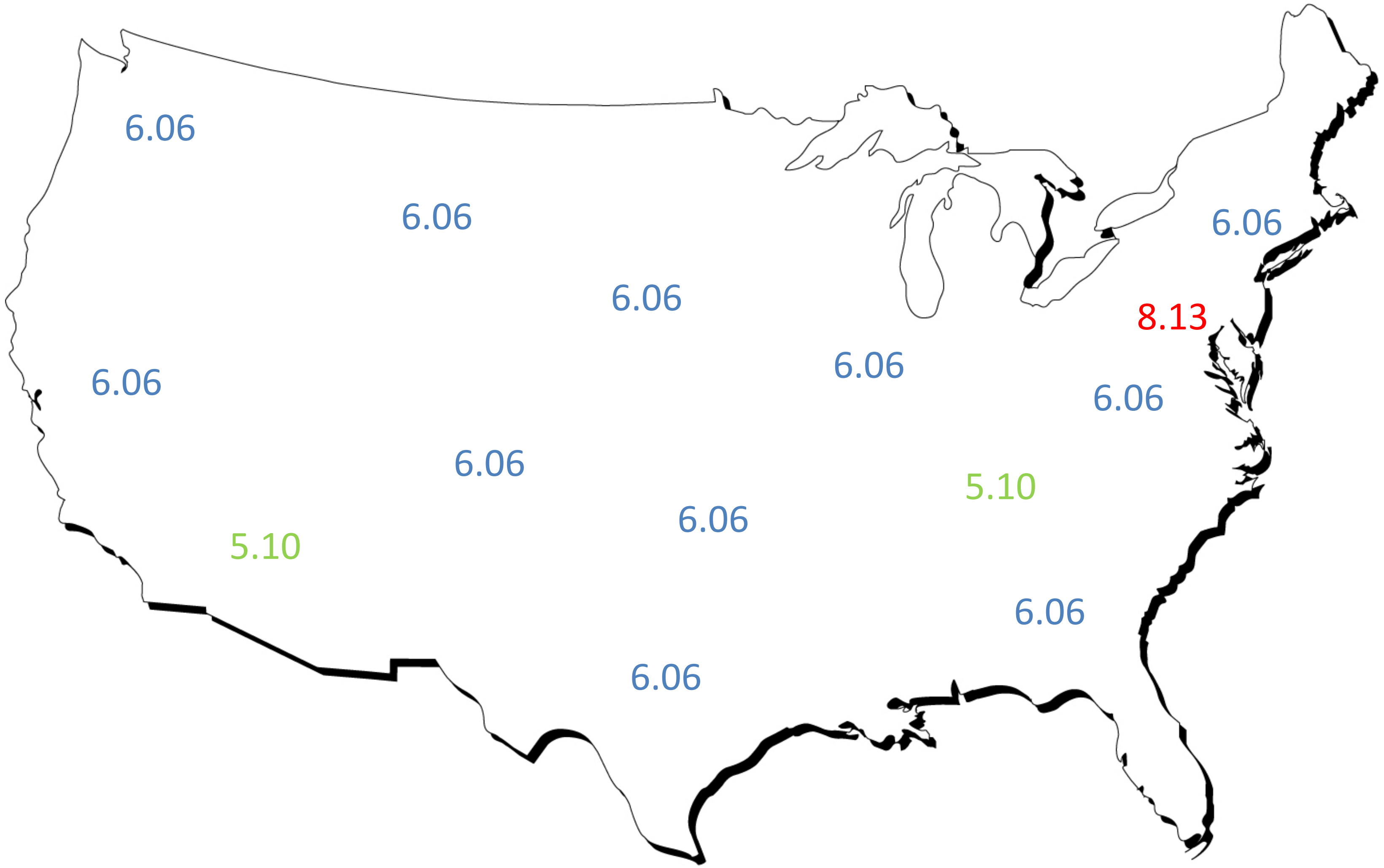


[1] Braga, A. A., Bond, B. J. (2008). Policing Crime and Disorder Hot Spots: A Randomized Controlled Trial. *Criminology, 46*(3), 577-607.

Motivation

Focusing on an easily-manageable subset of the system can yield enormous benefits for the system as a whole.

United States of America



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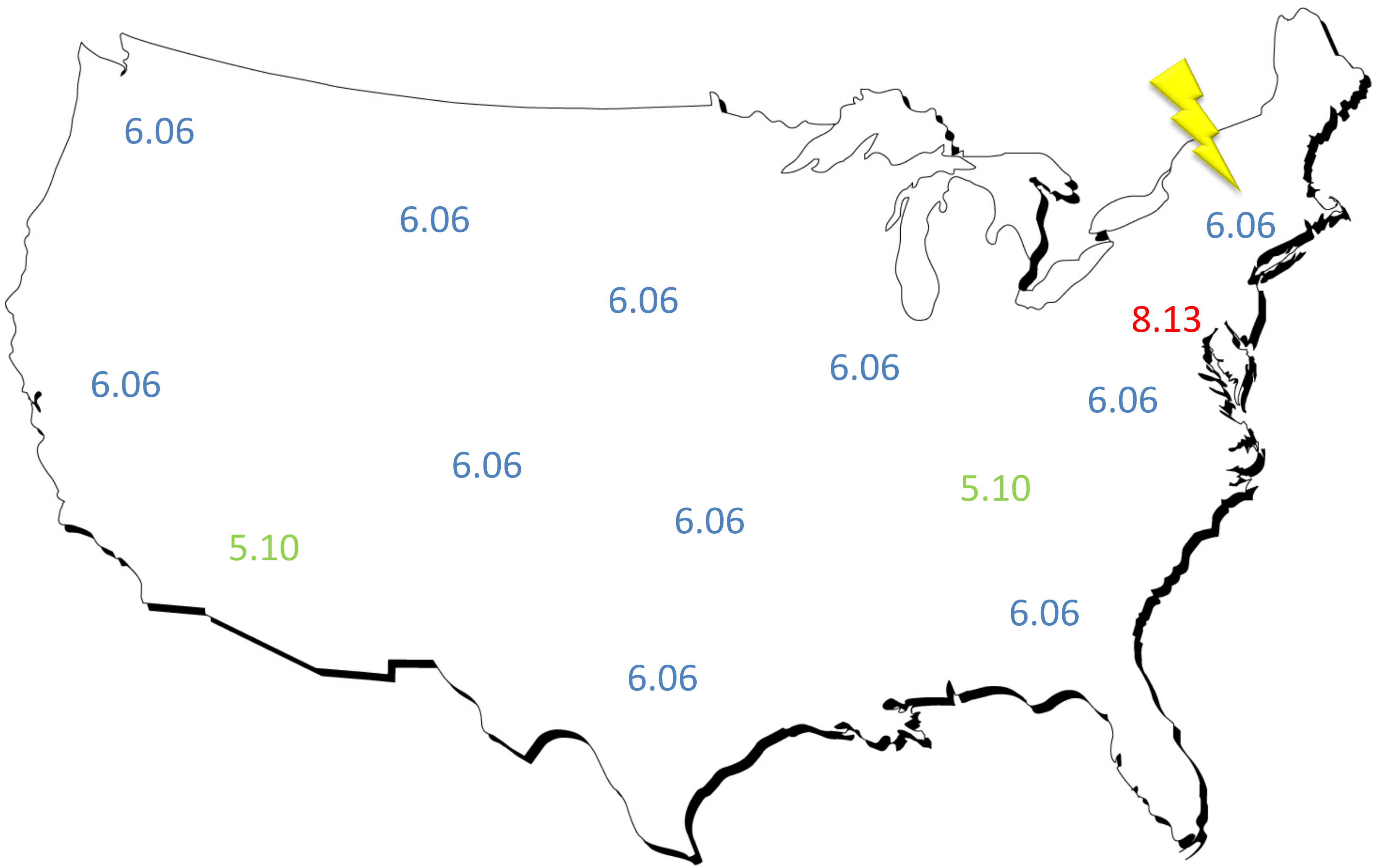
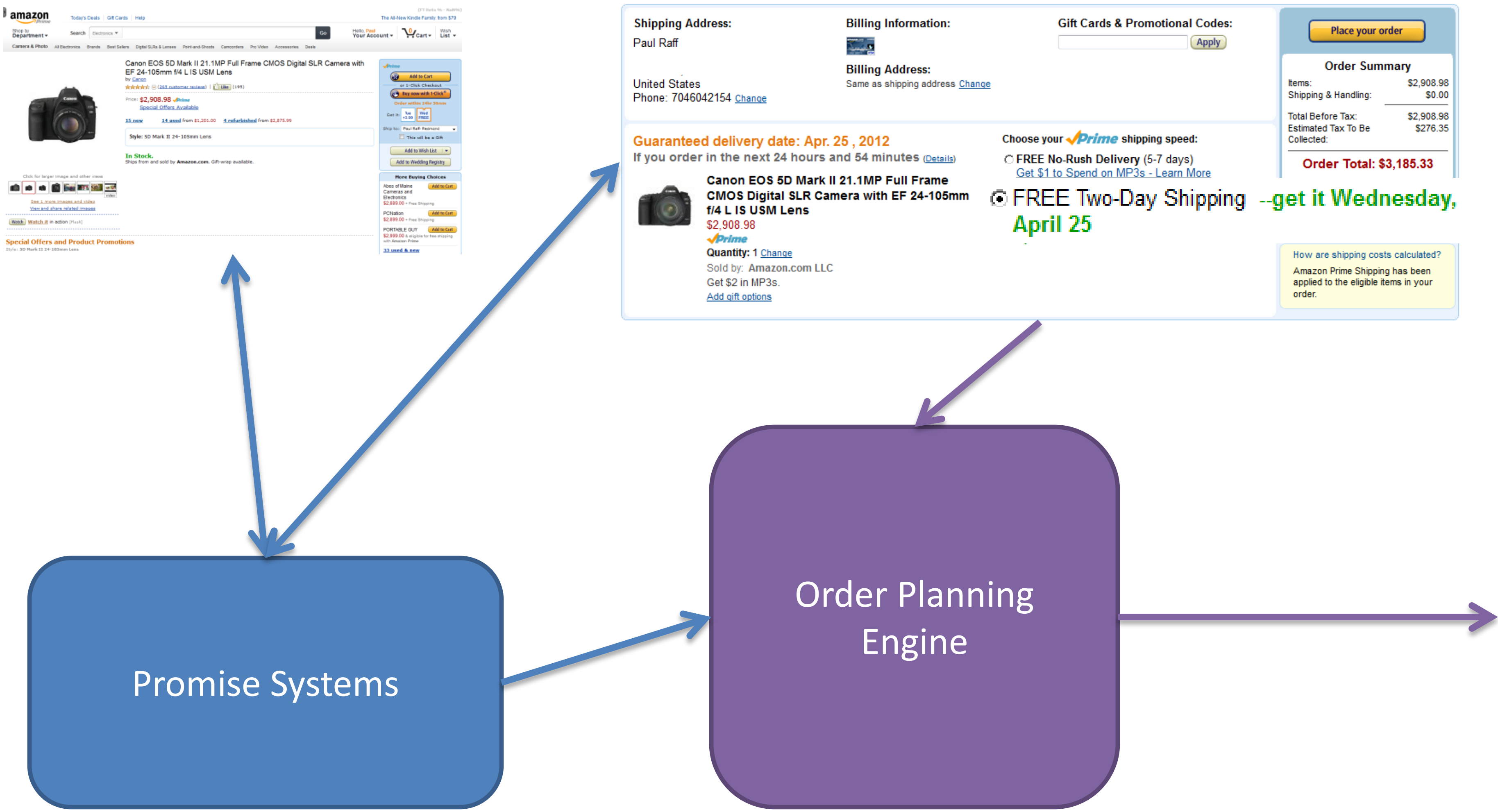


Overview: Amazon's Systems

(a very small slice)



The Core Problem

Location	Inventory	Cost: Super-saver to Seattle	Cost: Second-day to Seattle
PA	50	4.12	14.12
IN	1	3.87	6.12
SC	12	4.41	15.09
TN	15	4.89	11.51

- Our costs are determined by the order in which orders are placed.
  - Necessary at our scale.

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  - Necessary at our scale.
- If second-day order first, then total cost would be $10.24.
- If super-saver order first, then total cost would be $15.38.

A Real-Life Example

Location	Inventory	Cost: Order 1 03/20 00:01 PDT	Cost: Order 2 03/20 00:13 PDT
Warehouse 1	1	2.42	2.57
Warehouse 2	>1	2.56	3.87

A Real-Life Example

Location	Inventory	Cost: Order 1 03/20 00:01 PDT	Cost: Order 2 03/20 00:13 PDT
Warehouse 1	1	2.42	2.57
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A Subtle Modification

Location	Inventory	Cost: Order 1 03/20 00:01 PDT	Cost: Order 2 03/20 00:13 PDT
Warehouse 1	1	2.42	2.57
Warehouse 2	>1	2.42	3.87

Possible Solutions

- Have more inventory, or place it better.
  - Constantly worked on at Amazon
  - More inventory has a cost in itself
  - We can never always have perfect inventory placement

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  - Potential customer experience impact – how long do we wait?
- Opportunity cost
  - How to define?
  - Necessary inputs may be highly volatile or of dubious value

Note on Opportunity Cost

- Our order planning engine plans greedily, and hence sub-optimally.
- By having additional costs added to the solver, we can influence its decision:

FC	Fulfillment Cost	Opportunity Cost	Total Cost
RNO1	$3.50	$1.50	$5.00
LEX1	$4.00	$0.50	$4.50

- Opportunity cost calculates the amount we'd be willing to pay extra.

Note on Opportunity Cost

- Opportunity cost is a cost we *will* pay now for an *expectation* of savings in the future.
- Why pay more? A letter costs 46 cents to send, regardless of origin/destination.
- Would it suffice to only deal with tied situations?

Country	% of Units Involved In Tied Fulfillment Plans	
	Real-life	Optimally
US	16.04%	36.91%
DE	26.41%	49.23%

The Demand Model

Demand materializes sequentially, with the probability of demand coming from region i at any step being $p_i, \sum p_i = 1$.

<table>
<tr><td>Region 1, Region 1</td><td>0.64</td><td>0.64</td><td>Region 1: 2 units
Region 2: 0 units</td></tr>
<tr><td>Region 1, Region 2</td><td>0.16</td><td rowspan="2">0.32</td><td rowspan="2">Region 1: 1 unit
Region 2: 1 unit</td></tr>
<tr><td>Region 2, Region 1</td><td>0.16</td></tr>
<tr><td>Region 2, Region 2</td><td>0.04</td><td>0.04</td><td>Region 1: 0 units
Region 2: 2 units</td></tr>
</table>

A Note on Optimality

Finding the optimal solution for known demand and a given inventory level is not easy:

$$opt(\{d_1, d_2, \dots\} | \mathcal{I}) \\ = \min_{FC\ i}\left(c_{d_1,i} + opt(\{d_2, \dots\} \mid \mathcal{I} - e_i)\right)$$

Additionally, in a lot of cases it’s not practical.

Simple Example (no ties)

Situation:

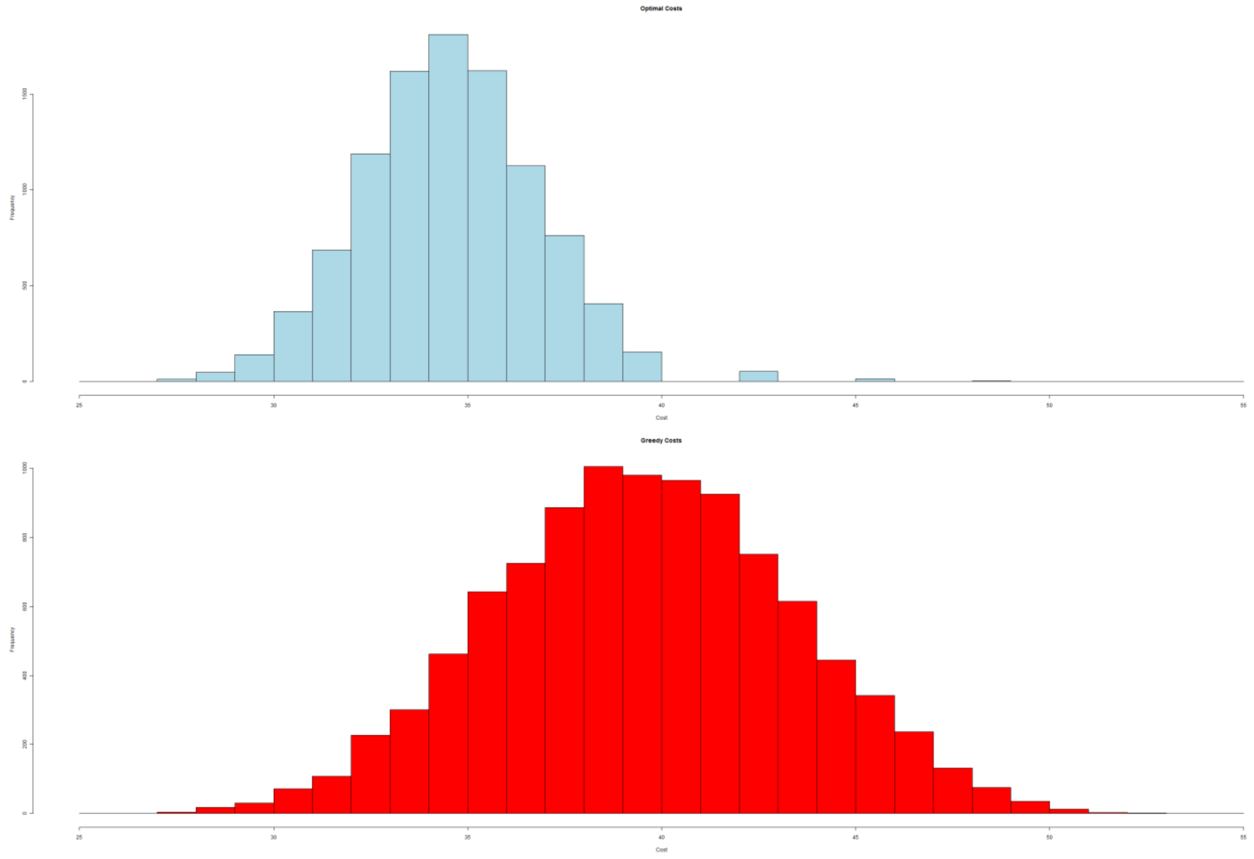
Order Type	Probability	Cost: Warehouse 1	Cost: Warehouse 2
Standard	50%	1	2
Express	50%	3	5

10,000 simulations of 20 orders, starting with inventory

	Warehouse 1	Warehouse 2
Inventory	15	5

Simple Example (no ties)

- On average, optimal saves $5 over greedy.



Typical Workaround

If we are dealing with orders greedily, then we will choose the warehouse that minimizes the following:

opportunity cost, or cost-to-go

$$c_{d,i} + f(\blacksquare),$$

where $\blacksquare$ can be a variety of things: inventory levels, past demand, forecasted demand, future expected inbound arrivals, . . .

Core Principle

- If we encounter an order that has tied fulfillment plans, we want to choose the option that has the *lowest future expected cost*.

- How do we calculate lowest future expected cost?

Single-SKU Demand Model

- Regions $R_1, R_2, \dots, R_n$
  - Demand probabilities $p_1, \dots, p_n$ with $\sum p_i = 1$
- Warehouses $W_1, W_2, \dots, W_m$
- Cost matrix showing the cost of fulfilling demand from W_i to region R_j

Order Type	Probability	Cost: Warehouse 1	Cost: Warehouse 2
Super Saver	50%	1	2
Express	50%	3	5

Simplest Recursion

- Two warehouses, two regions

Region	Probability	Cost: Warehouse 1		Cost: Warehouse 2
R_1	p_1	c_{11}	$<$	c_{12}
R_2	p_2	c_{21}	$>$	c_{22}

$$cost(0, b) = p_1 c_{12} b + p_2 c_{22} b$$

$$cost(a, 0) = p_1 c_{11} a + p_2 c_{21} a$$

$$cost(a, b) = \begin{array}{ll} & p_1(c_{11} + cost(a-1, b)) \\ + & p_2(c_{22} + cost(a, b-1)) \end{array}$$

What About Ties?

Region	Probability	Cost: Warehouse 1		Cost: Warehouse 2
R_1	p_1	c_{11}	$<$	c_{12}
R_2	p_2	c_{21}	$>$	c_{22}
R_3	p_3	c_{31}	$=$	c_{32}

$$cost(a,b) = \begin{aligned} & p_1(c_{11} + cost(a-1,b)) \\ + \quad & p_2(c_{22} + cost(a,b-1)) \\ + \quad & p_3(c_{31} + ?) \end{aligned}$$

What About Ties?

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R_3	p_3	c_{31}	$=$	c_{32}

$$cost(a,b) = \begin{array}{ll} & p_1(c_{11} + cost(a-1,b)) \\ + & p_2(c_{22} + cost(a,b-1)) \\ + & p_3\left(c_{31} + \min\begin{cases} cost(a-1,b) \\ cost(a,b-1) \end{cases}\right) \end{array}$$

What About 3 Warehouses?

$W_1 < W_2 < W_3$	$W_1 = W_2 < W_3$
$W_1 < W_3 < W_2$	$W_3 < W_1 = W_2$
$W_2 < W_1 < W_3$	$W_1 = W_3 < W_2$
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$W_1 = W_2 = W_3$	

What About 4 Warehouses?

- 75 different possibilities!
- With 50 warehouses, there are 1,995,015,910,118,319,790,635,433,747,742,913,123,711,612,309,013,079,035,980,385,090,523,556,363 possibilities† !
- In practice, Amazon.com observed ~250K different scenarios in its NA network.

† OEIS A000670

Common Topologies

Common Topologies

In Practice

Region	Probability	Cost: Warehouse 1	Cost: Warehouse 2
R_1	40%	1	2
R_2	40%	3	1
R_3	20%	2	2
		↓	
R_1	40%	0	1
R_2	40%	2	0
R_3	20%	0	0

In Practice

Region	Probability	Cost: Warehouse 1	Cost: Warehouse 2
R_1	40%	0	1
$\boldsymbol{R_2}$	**20%**	**1**	**0**
$\boldsymbol{R_3}$	**20%**	**3**	**0**
R_4	20%	0	0
	↓		
R_1	40%	0	1
$\boldsymbol{R_2}$	**40%**	**2**	**0**
R_3	20%	0	0

Region	Probability	Cost: Warehouse 1	Cost: Warehouse 2
R_1	40%	0	1
R_2	40%	2	0
R_3	20%	0	0

20	8.00	7.40	6.80	6.20	5.61	5.02	4.46	3.92	3.42	2.97	2.58	2.25	1.99	1.79	1.65	1.56	1.51	1.48	1.48	1.49	1.51
19	7.60	7.00	6.40	5.80	5.21	4.63	4.08	3.56	3.08	2.67	2.32	2.03	1.82	1.66	1.56	1.50	1.47	1.47	1.48	1.50	1.54
18	7.20	6.60	6.00	5.41	4.82	4.25	3.71	3.21	2.77	2.39	2.08	1.85	1.67	1.56	1.49	1.46	1.45	1.46	1.49	1.54	1.61
17	6.80	6.20	5.60	5.01	4.43	3.87	3.35	2.88	2.47	2.14	1.88	1.69	1.56	1.48	1.45	1.44	1.45	1.48	1.53	1.61	1.72
16	6.40	5.80	5.20	4.61	4.04	3.50	3.00	2.56	2.20	1.91	1.70	1.56	1.47	1.43	1.42	1.43	1.46	1.52	1.61	1.73	1.89
15	6.00	5.40	4.80	4.22	3.66	3.13	2.67	2.27	1.96	1.72	1.56	1.47	1.42	1.40	1.42	1.45	1.51	1.60	1.74	1.91	2.13
14	5.60	5.00	4.41	3.83	3.28	2.79	2.36	2.01	1.75	1.57	1.46	1.40	1.38	1.40	1.43	1.50	1.60	1.74	1.93	2.17	2.45
13	5.20	4.60	4.01	3.44	2.92	2.46	2.07	1.78	1.57	1.45	1.38	1.36	1.37	1.41	1.49	1.60	1.75	1.96	2.22	2.52	2.88
12	4.80	4.20	3.62	3.06	2.57	2.15	1.82	1.58	1.44	1.36	1.34	1.35	1.39	1.47	1.59	1.77	1.99	2.27	2.60	2.99	3.42
11	4.40	3.80	3.23	2.70	2.24	1.87	1.60	1.43	1.34	1.31	1.32	1.37	1.46	1.59	1.78	2.02	2.32	2.68	3.10	3.56	4.08
10	4.00	3.41	2.84	2.34	1.93	1.62	1.42	1.31	1.28	1.29	1.34	1.44	1.59	1.79	2.06	2.39	2.78	3.22	3.72	4.26	4.85
9	3.60	3.01	2.47	2.01	1.65	1.42	1.29	1.24	1.26	1.31	1.42	1.58	1.81	2.10	2.46	2.88	3.36	3.89	4.46	5.08	5.73
8	3.20	2.62	2.11	1.70	1.42	1.26	1.20	1.21	1.27	1.39	1.58	1.83	2.15	2.54	3.00	3.51	4.07	4.68	5.33	6.00	6.71
7	2.80	2.24	1.76	1.42	1.23	1.15	1.16	1.23	1.36	1.57	1.85	2.21	2.64	3.13	3.68	4.28	4.92	5.59	6.30	7.02	7.76
6	2.40	1.86	1.45	1.20	1.10	1.10	1.18	1.33	1.56	1.88	2.28	2.75	3.28	3.87	4.50	5.18	5.88	6.60	7.35	8.11	8.88
5	2.00	1.50	1.17	1.03	1.03	1.11	1.29	1.55	1.91	2.36	2.87	3.45	4.08	4.75	5.46	6.18	6.93	7.69	8.47	9.25	10.03
4	1.60	1.16	0.96	0.94	1.03	1.23	1.55	1.96	2.45	3.02	3.65	4.32	5.03	5.76	6.51	7.28	8.05	8.84	9.62	10.42	11.21
3	1.20	0.87	0.82	0.93	1.17	1.54	2.01	2.57	3.20	3.88	4.59	5.33	6.09	6.86	7.64	8.43	9.22	10.01	10.81	11.60	12.40
2	0.80	0.66	0.78	1.09	1.54	2.09	2.72	3.41	4.14	4.89	5.66	6.44	7.22	8.01	8.81	9.61	10.40	11.20	12.00	12.80	13.60
1	0.40	0.56	0.98	1.55	2.21	2.92	3.67	4.44	5.23	6.02	6.81	7.61	8.40	9.20	10.00	10.80	11.60	12.40	13.20	14.00	14.80
0	0.00	0.80	1.60	2.40	3.20	4.00	4.80	5.60	6.40	7.20	8.00	8.80	9.60	10.40	11.20	12.00	12.80	13.60	14.40	15.20	16.00
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**

Region	Probability	Cost: Warehouse 1	Cost: Warehouse 2
R_1	40%	0	1
R_2	40%	10	0
R_3	20%	0	0

20	8.00	7.40	6.80	6.20	5.62	5.05	4.50	4.01	3.57	3.20	2.92	2.71	2.57	2.50	2.47	2.48	2.52	2.60	2.74	2.96	3.26
19	7.60	7.00	6.40	5.81	5.22	4.66	4.14	3.68	3.28	2.96	2.73	2.57	2.48	2.45	2.46	2.50	2.58	2.73	2.95	3.27	3.71
18	7.20	6.60	6.00	5.41	4.84	4.29	3.79	3.36	3.01	2.75	2.57	2.46	2.42	2.43	2.47	2.55	2.71	2.95	3.29	3.76	4.36
17	6.80	6.20	5.60	5.02	4.45	3.93	3.46	3.08	2.78	2.57	2.45	2.39	2.40	2.43	2.52	2.69	2.94	3.31	3.80	4.46	5.28
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15	6.00	5.40	4.81	4.24	3.71	3.25	2.87	2.60	2.42	2.34	2.32	2.36	2.45	2.63	2.92	3.33	3.91	4.66	5.60	6.75	8.13
14	5.60	5.00	4.42	3.86	3.36	2.94	2.62	2.41	2.30	2.28	2.31	2.41	2.60	2.90	3.35	3.96	4.77	5.78	7.01	8.48	10.18
13	5.20	4.60	4.02	3.49	3.02	2.66	2.41	2.27	2.23	2.26	2.36	2.55	2.88	3.36	4.02	4.88	5.97	7.30	8.87	10.68	12.74
12	4.80	4.21	3.64	3.13	2.71	2.42	2.24	2.18	2.20	2.30	2.51	2.85	3.36	4.07	5.01	6.18	7.61	9.29	11.22	13.41	15.82
11	4.40	3.81	3.26	2.79	2.44	2.22	2.13	2.14	2.24	2.45	2.81	3.36	4.13	5.14	6.41	7.94	9.75	11.81	14.13	16.68	19.45
10	4.00	3.42	2.89	2.48	2.20	2.07	2.07	2.16	2.38	2.76	3.36	4.19	5.28	6.65	8.31	10.24	12.45	14.91	17.60	20.51	23.60
9	3.60	3.03	2.54	2.20	2.02	1.99	2.07	2.29	2.70	3.34	4.25	5.44	6.92	8.71	10.79	13.15	15.76	18.60	21.64	24.86	28.23
8	3.20	2.65	2.22	1.97	1.90	1.97	2.19	2.63	3.32	4.30	5.60	7.22	9.15	11.39	13.91	16.68	19.67	22.86	26.21	29.69	33.28
7	2.80	2.28	1.93	1.80	1.85	2.07	2.53	3.28	4.36	5.78	7.55	9.65	12.06	14.75	17.69	20.84	24.16	27.64	31.23	34.91	38.68
6	2.40	1.93	1.70	1.71	1.92	2.41	3.22	4.41	5.97	7.91	10.20	12.80	15.68	18.79	22.09	25.56	29.15	32.84	36.61	40.44	44.32
5	2.00	1.62	1.55	1.74	2.24	3.13	4.45	6.18	8.32	10.82	13.63	16.70	19.99	23.46	27.05	30.76	34.54	38.38	42.27	46.19	50.13
4	1.60	1.36	1.50	2.02	3.00	4.47	6.41	8.79	11.52	14.56	17.84	21.31	24.93	28.65	32.45	36.31	40.21	44.15	48.10	52.07	56.05
3	1.20	1.21	1.70	2.79	4.47	6.68	9.33	12.34	15.63	19.12	22.77	26.52	30.35	34.23	38.16	42.10	46.07	50.04	54.03	58.02	62.01
2	0.80	1.22	2.45	4.42	6.99	9.99	13.31	16.86	20.56	24.36	28.23	32.15	36.09	40.06	44.04	48.02	52.02	56.01	60.01	64.00	68.00
1	0.40	1.84	4.30	7.38	10.83	14.50	18.30	22.18	26.11	30.06	34.04	38.02	42.01	46.01	50.01	54.00	58.00	62.00	66.00	70.00	74.00
0	0.00	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00	60.00	64.00	68.00	72.00	76.00	80.00
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**

In Practice

Inventory		3	6
Region	Probability	Cost: Warehouse 1	Cost: Warehouse 2
R_1	40%	0	1
R_2	40%	10	0
R_3	20%	0	0

Inventory in FC 2								
	6	2.40	1.93	1.70	**1.71**	1.92	2.41	3.22
	5	2.00	1.62	1.55	1.74	2.24	3.13	4.45
	4	1.60	1.36	1.50	2.02	3.00	4.47	6.41
	3	1.20	1.21	1.70	2.79	4.47	6.68	9.33
	2	0.80	1.22	2.45	4.42	6.99	9.99	13.31
	1	0.40	1.84	4.30	7.38	10.83	14.50	18.30
	0	0.00	4.00	8.00	12.00	16.00	20.00	24.00
		0	**1**	**2**	**3**	**4**	**5**	**6**
		Inventory in FC 1						

What Does It Mean?

- We don’t want to be left with inventory only in Warehouse 1.

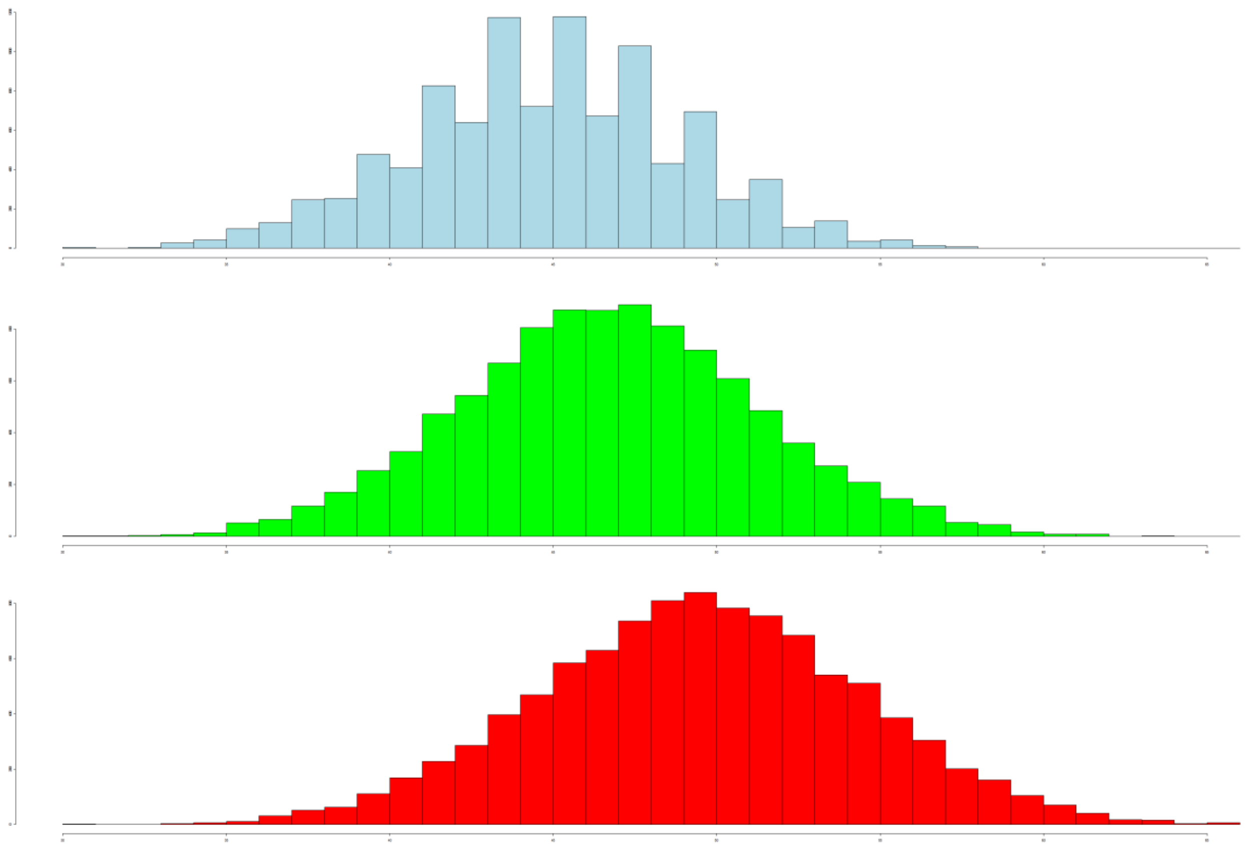
20	8.00	7.40	6.80	6.20	5.62	5.05	4.50	4.01	3.57	3.20	2.92	2.71	2.57	2.50	2.47	2.48	2.52	2.60	2.74	2.96	3.26
19	7.60	7.00	6.40	5.81	5.22	4.66	4.14	3.68	3.28	2.96	2.73	2.57	2.48	2.45	2.46	2.50	2.58	2.73	2.95	3.27	3.71
18	7.20	6.60	6.00	5.41	4.84	4.29	3.79	3.36	3.01	2.75	2.57	2.46	2.42	2.43	2.47	2.55	2.71	2.95	3.29	3.76	4.36
17	6.80	6.20	5.60	5.02	4.45	3.93	3.46	3.08	2.78	2.57	2.45	2.39	2.40	2.43	2.52	2.69	2.94	3.31	3.80	4.46	5.28
16	6.40	5.80	5.21	4.63	4.08	3.58	3.15	2.82	2.58	2.43	2.37	2.36	2.40	2.49	2.66	2.93	3.32	3.85	4.55	5.43	6.51
15	6.00	5.40	4.81	4.24	3.71	3.25	2.87	2.60	2.42	2.34	2.32	2.36	2.45	2.63	2.92	3.33	3.91	4.66	5.60	6.75	8.13
14	5.60	5.00	4.42	3.86	3.36	2.94	2.62	2.41	2.30	2.28	2.31	2.41	2.60	2.90	3.35	3.96	4.77	5.78	7.01	8.48	10.18
13	5.20	4.60	4.02	3.49	3.02	2.66	2.41	2.27	2.23	2.26	2.36	2.55	2.88	3.36	4.02	4.88	5.97	7.30	8.87	10.68	12.74
12	4.80	4.21	3.64	3.13	2.71	2.42	2.24	2.18	2.20	2.30	2.51	2.85	3.36	4.07	5.01	6.18	7.61	9.29	11.22	13.41	15.82
11	4.40	3.81	3.26	2.79	2.44	2.22	2.13	2.14	2.24	2.45	2.81	3.36	4.13	5.14	6.41	7.94	9.75	11.81	14.13	16.68	19.45
10	4.00	3.42	2.89	2.48	2.20	2.07	2.07	2.16	2.38	2.76	3.36	4.19	5.28	6.65	8.31	10.24	12.45	14.91	17.60	20.51	23.60
9	3.60	3.03	2.54	2.20	2.02	1.99	2.07	2.29	2.70	3.34	4.25	5.44	6.92	8.71	10.79	13.15	15.76	18.60	21.64	24.86	28.23
8	3.20	2.65	2.22	1.97	1.90	1.97	2.19	2.63	3.32	4.30	5.60	7.22	9.15	11.39	13.91	16.68	19.67	22.86	26.21	29.69	33.28
7	2.80	2.28	1.93	1.80	1.85	2.07	2.53	3.28	4.36	5.78	7.55	9.65	12.06	14.75	17.69	20.84	24.16	27.64	31.23	34.91	38.68
6	2.40	1.93	1.70	1.71	1.92	2.41	3.22	4.41	5.97	7.91	10.20	12.80	15.68	18.79	22.09	25.56	29.15	32.84	36.61	40.44	44.32
5	2.00	1.62	1.55	1.74	2.24	3.13	4.45	6.18	8.32	10.82	13.63	16.70	19.99	23.46	27.05	30.76	34.54	38.38	42.27	46.19	50.13
4	1.60	1.36	1.50	2.02	3.00	4.47	6.41	8.79	11.52	14.56	17.84	21.31	24.93	28.65	32.45	36.31	40.21	44.15	48.10	52.07	56.05
3	1.20	1.21	1.70	2.79	4.47	6.68	9.33	12.34	15.63	19.12	22.77	26.52	30.35	34.23	38.16	42.10	46.07	50.04	54.03	58.02	62.01
2	0.80	1.22	2.45	4.42	6.99	9.99	13.31	16.86	20.56	24.36	28.23	32.15	36.09	40.06	44.04	48.02	52.02	56.01	60.01	64.00	68.00
1	0.40	1.84	4.30	7.38	10.83	14.50	18.30	22.18	26.11	30.06	34.04	38.02	42.01	46.01	50.01	54.00	58.00	62.00	66.00	70.00	74.00
0	0.00	4.00	8.00	12.00	16.00	20.00	24.00	28.00	32.00	36.00	40.00	44.00	48.00	52.00	56.00	60.00	64.00	68.00	72.00	76.00	80.00
	0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**

Simple Example (with ties)

Order Type	Probability	Cost: Warehouse 1	Cost: Warehouse 2
Super Saver	40%	1	2
Express	40%	3	5
Standard	20%	3	3
Initial inventory: (15, 5)			

Simple Example (with ties)

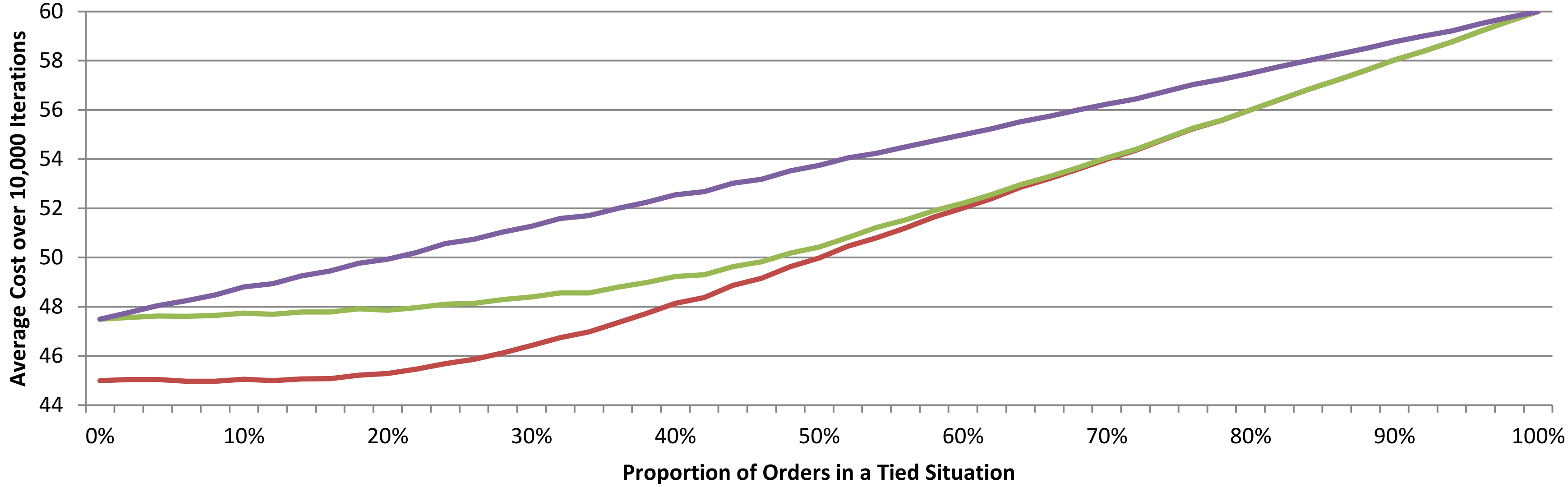
Scenario	Optimal	Tie-breaking	Greedy
Average Fulfillment Cost	45.29	47.26	49.97



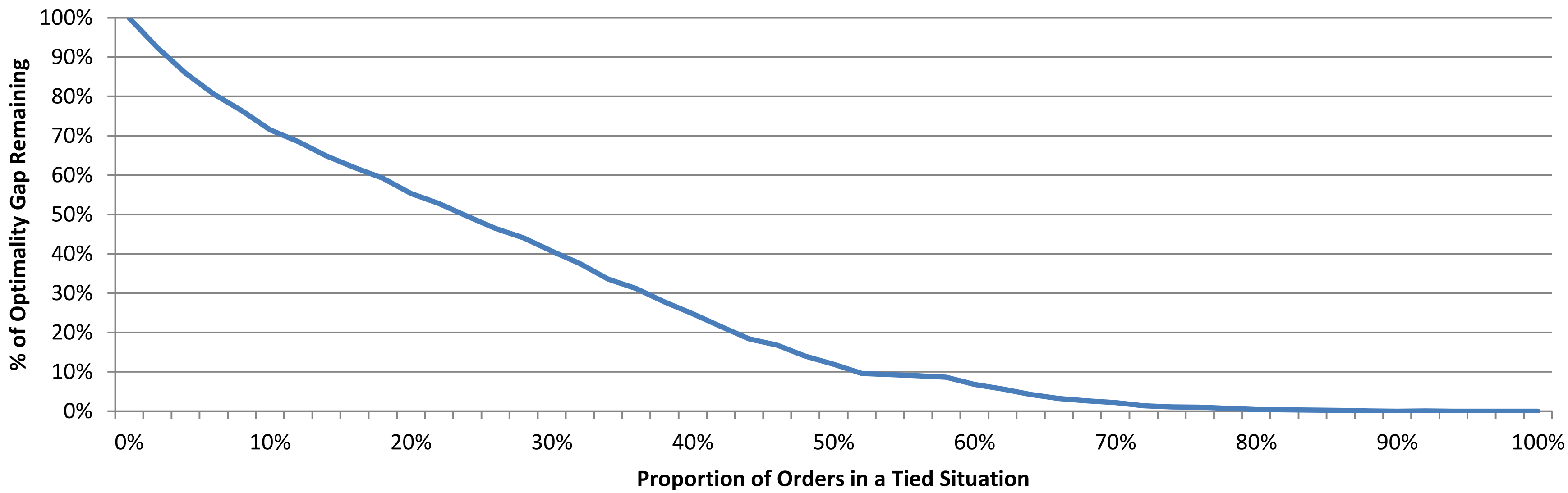
Simple Example (with ties)

Order Type	Probability	Cost: Warehouse 1	Cost: Warehouse 2
Super Saver	$\frac{100-x}{2}\%$	1	2
Express	$\frac{100-x}{2}\%$	3	5
Standard	$x\%$	3	3
Initial Inventory: (15, 5)			

Tie-Breaking Proportion over Varying Proportions of Tied Orders



Optimality Gap over Varying Proportions of Tied Orders (relative)



Results

- Via Amazon’s supply chain simulator:
  – DE: covered 22% of optimality gap
  – JP: covered 60% of optimality gap
  – 75% of savings via split-shipment reduction
- Results validated via controlled experiment in production.

Future Steps

- Model modifications
  - Primary addition: pending arrival of purchase orders
  - Initial experiments indicated very little improvement even with perfect knowledge of PO arrivals
- Back-door to opportunity cost
- Applications to other areas of supply chain